VERTEX COLORING OF DOUBLE LAYERED COMPLETE FUZZY GRAPH USING α- CUT

M. Anandhi

Assistant Professor, PG and Research Department of Mathematics, Theivanai Ammal College for Women (Autonomous), Villupuram – 605 602, Tamil Nadu, India.

B. Keerthiga

II M.Sc Mathematics, PG and Research Department of Mathematics, Theivanai Ammal College for Women (Autonomous), Villupuram – 605 602, Tamil Nadu, India.

ABSTRACT

In this paper, we defined a new fuzzy graph named double layered complete fuzzy graph (DLCFG).The double layered complete fuzzy graph gives a 3- D structure. Further we introduced the vertex coloring on double layered complete fuzzy graph using α- cut.

KEYWORDS: Fuzzy graph, complete fuzzy graph, alpha cut, DLFG, DLCFG, coloring of DLFG,Coloring of DLCFG using alpha cut.

1.INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975.Graph theory is proved to be tremendously useful in modeling the essential features of systems with finite components. If the relation among accounts are to be measured as good or bad according to the frequency of contacts among the accounts,fuzziness should be added to representation. This and many another problem motivated to define fuzzy graphs Rosenfeld first introduced the concept of fuzzy graph. After that fuzzy graph theory becomes a vast researched area.A fuzzy set is defined mathematically by assigning to each possible individual in the universe of discourse a value , representing its grade of membership , which corresponds to the degree, to which that individual is similar or compatible with the concept represented by the fuzzy set. Fuzzy graphs have many more applications in modeling real time systems where the level of information inherent in the systems varies with different level of precision.

In this paper ,The notion of fuzzy set which is characterized by a membership function which assigns to each object a grade of membership which ranges from 0 to 1. The fuzzy coloring problem consists of determining the chromatic number of a fuzzy graph and an associated coloring function. For any level Alpha , the minimum number of colors needed to color the crisp graph G α will be computed. In this way the fuzzy chromatic number is defined as fuzzy number through its α – cuts.

1. PRELIMINARIES

Definition: Fuzzy Graph

A **fuzzy graph** $G = (\sigma,\mu)$ is a pair of function $\sigma: V\rightarrow [0,1]$ and $\mu: V\times V\rightarrow[0,1]$, where for all $x,y\in V$, we have $\mu(x,y) \leq \sigma(x)\wedge \sigma(y)$.

Example

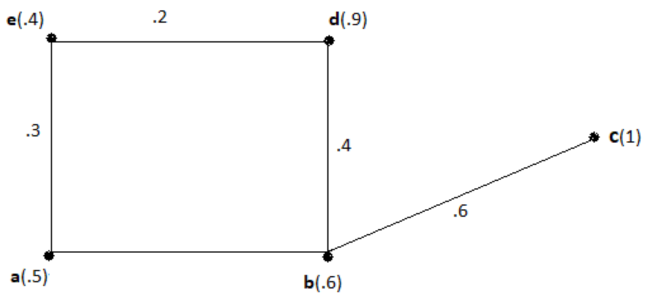


Figure 1

Definition: Order and size

The **order and size** of a fuzzy graph G=(σ,μ) are defined to be $\sum_{x\epsilon v}\sigma(x)$and. $\sum_{xy\epsilon E}\mu(xy)$It is also denoted as O(G) and S(G).

Definition: Degree of a vertex

Let G = (σ,μ) be a fuzzy graph. The **degree of a vertex** u is defined asd (u) =$\sum_{u,v\epsilon G}\mu(u,v)$. It is also denoted as$d_G$(u).

Definition: Complete fuzzy graph

Let σ:$V\rightarrow$ [0,1] be a fuzzy subset of V then the **complete fuzzy graph** on σ is defined on

G= (σ, μ) whereμ(xy)=σ(x) ∧σ(y) ∀x, y∈E. It is denoted by k_σ.

Example

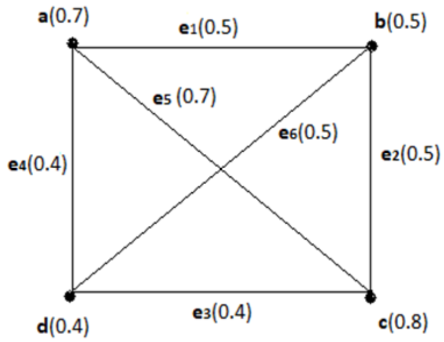


Figure 2

Definition: Adjacent

Two vertices $u$and $v$for any strong edge in $G$are called **adjacent** if

(1/2) min {σ (u),σ(v)} ≤μ (uv).

Example

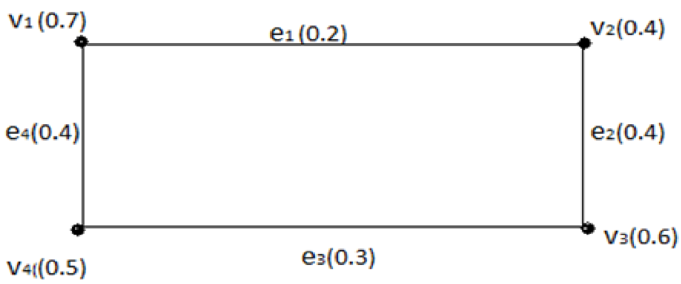


Figure 3

Definition: Incident

Two edges $v_i v_j$and $v_j v_k$are said to be **incident** if,

2 min {μ($v_i v_j$),μ($v_j v_k$)}≤σ(v_j) $forj$ = 1,2…|v| ,1 ≤i,k≤ |v|.

Example

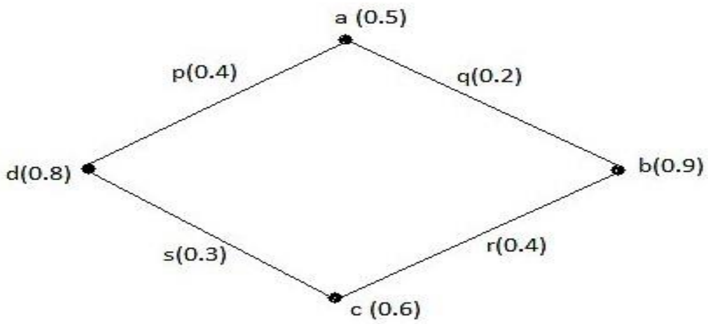


Figure 4

Definition : $\alpha - cut$

The **α cut of fuzzy graph** defined as $G_\alpha=(V_\alpha, E_\alpha)$ where $V_\alpha=\{v\in V/ \sigma \geq \alpha\}$ and

$E_\alpha = \{e\in E /\mu \geq\alpha\}$.

Definition :K-coloring fuzzy graph

A family $\Gamma = \{\gamma_1, \gamma_2, \ldots \gamma_k\}$ of fuzzy sets on V is called a ***k*-coloring of fuzzy graph** $G = (V,\sigma,\mu)$

(a) $\vee\Gamma = \sigma$,

(b) $\gamma_i \wedge \gamma_j = 0$

(c) For every strong edge xy of G, $min\ \{(\gamma_i(x), \gamma_i(y)\} = 0\ (1\leq i \leq k)$

The least value of k for which G has a k-fuzzy coloring denoted by $\chi_F(G)$, is called the fuzzy chromatic number of G

Example

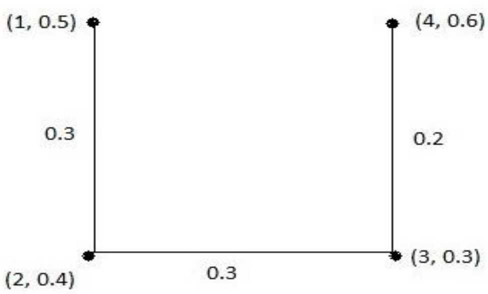


Figure 5

Definition : Double layered Fuzzy Graph

Let G=(σ, μ) be a fuzzy graph with the underlying crisp graph G* = (σ* , μ*). The pair

DL(G) = (σ_{DL}, μ_{DL}) is defined as follow. The node set of DL(G) be σ* ∪μ*. The fuzzy subset

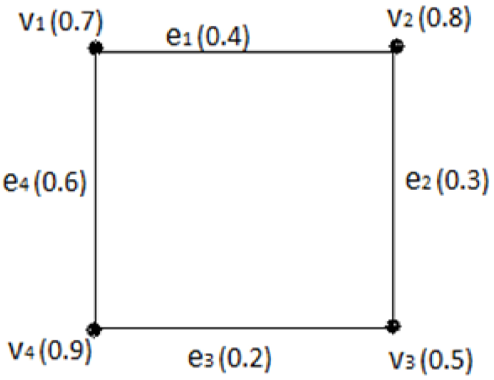
$$\sigma_{DL} = \begin{cases} \sigma(u) \text{ if } u \in \sigma * \\ \mu(uv) \text{ if } uv \in \mu * \end{cases}$$

The fuzzy relation μ_{DL} on σ* ∪ μ* is defined as

$$\mu_{DL} = \begin{cases} \mu(uv) \text{ if } u, v \in \sigma * \\ \mu(e) \wedge \mu(e) \text{ if the edge e and e have a node in common between them} \\ \sigma(u) \wedge \mu(e) \text{ if } u \in \sigma * \text{ and } e \in \mu * \text{ and each e is incident with single u either clockwise or anticlockwise} \\ 0 \text{ otherwise} \end{cases}$$

Example: Consider the fuzzy graph G, where crisp graph G* is a cycle with n=4 vertices

Figure 6



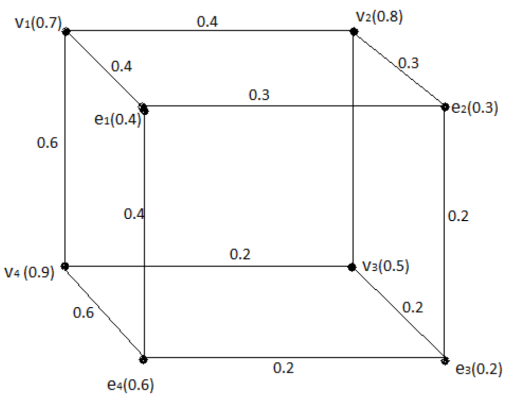


Figure 7

Some of the properties using Order and Size

- Order DL(G) = Order (G) + Size (G) where G is a fuzzy graph
- Size DL(G)= 2 Size (G) +$\sum_{ei,ej\epsilon E} \mu(e_i) \wedge \mu(e_j)$
- | EDL(G)| = 2 |E(L(G))|

DOUBLE LAYERED COMPLETE FUZZY GRAPH(DLCFG)

Definition:

Let σ_{DL}: V→ [0, 1] be a fuzzy subset of V then the complete double layered fuzzy graph onσ_{DL} is defined on $K_{\sigma \cup \mu}$= (σ_{DL}, μ_{DL}). Any two vertices of the DLCFG is adjacent.

Example-1:

Consider the complete fuzzy graph with vertex 3 (K3)

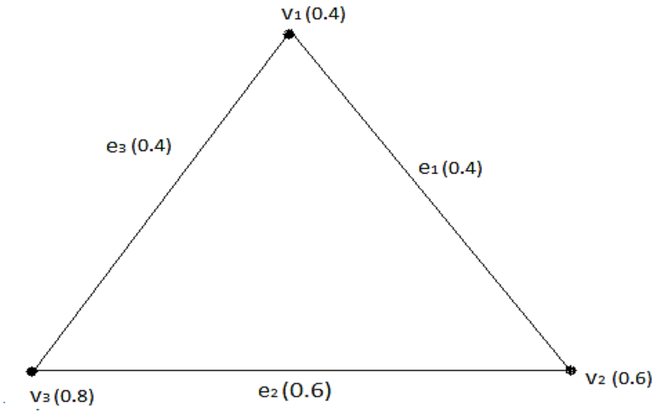


Figure 8

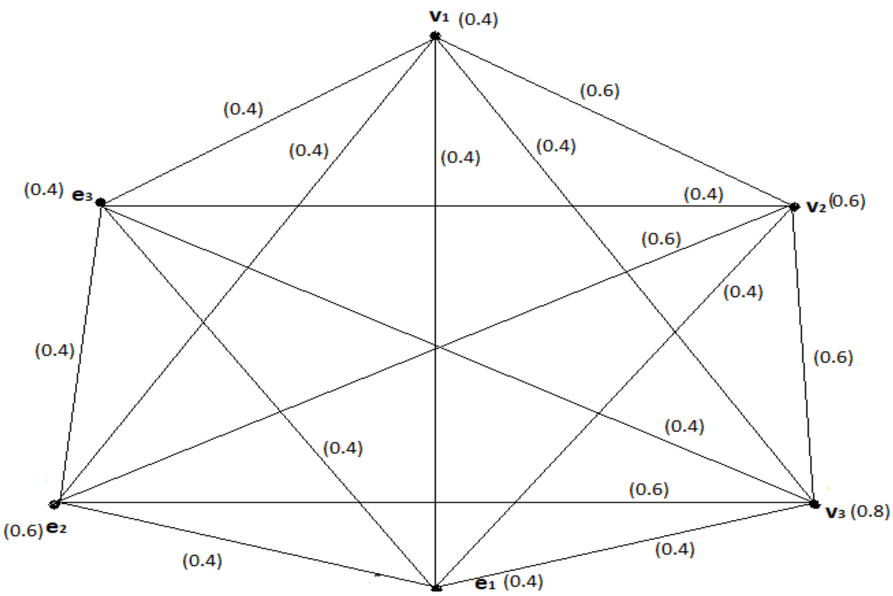


Figure 9

VERTEX COLORING OF DOUBLE LAYERED COMPLETE FUZZY GRAPH USING α- CUT

In this DLCFG (fig 9),there are 3 Alpha -cut is presented, They are {0.4, 0.6, 0.8}.For every value of Alpha, we find DLCF(G_α) and find its fuzzy chromatic number. For **α=0.4** Double layered complete fuzzy graph DLC(G)=(σ_{DL},μ_{DL}) Where

σ_{DLC} ={0.8, 0.6,0.4}

Here We need minimum 6 colors to proper color all the vertices of the graph DLC($G_{0.4}$).so the chromatic number of DLC ($G_{0.4}$) is 6

For α= 0.4 ,$\chi_{0.4}$ = χ_{DL}(0.4) = 6

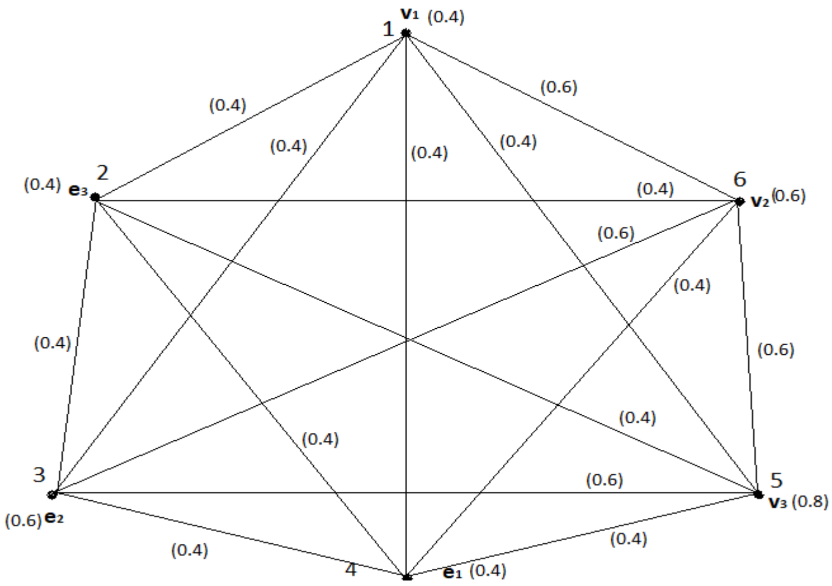


Figure 10

For **α=0.6** Double layered complete fuzzy graph DLC(G)=(σ_{DL},μ_{DL}) Where

σ_{DLC}={0.8, 0.6}

Here We need minimum 2 colors to proper color all the vertices of the graph DLC($G_{0.6}$).so the chromatic number of DLC ($G_{0.6}$) is 2

For α= 0.6 ,$\chi_{0.6}$ = χ_{DL}(0.6) = 2

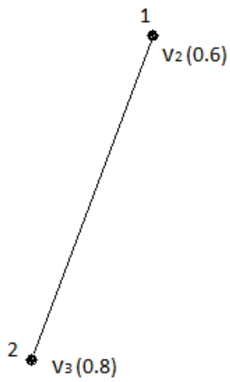


Figure 11

For **α=0.8** Double layered complete fuzzy graph DLC(G)=(σ_{DL},μ_{DL}) Where

σ_{DLC}={0.8}

Here We need minimum 1 colors to proper color all the vertices of the graph DLC($G_{0.8}$).so the chromatic number of DLC ($G_{0.8}$) is 1

For α= 0.8 ,$\chi_{0.8}$ = χ_{DL}(0.8) = 1

1 $v_{3(0.8)}$

Figure 12

Example-2:

Consider the complete fuzzy graph with vertex 4 (K4)

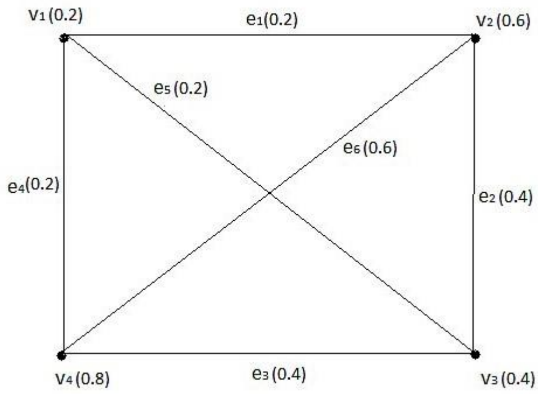


Figure 13

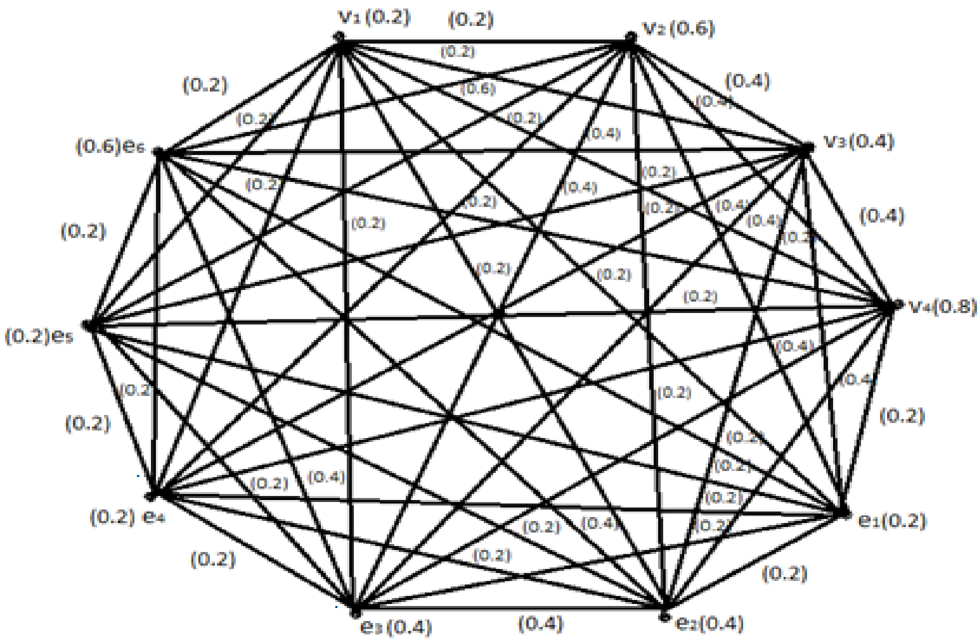


Figure 14

In this DLCFG, there are 4 Alpha -cut is presented,

They are {0.2, 0.4, 0.6,0.8}. For every value of Alpha, we find DLCF(G_α) and find its fuzzy chromatic number. For **α=0.2** Double layered complete fuzzy graph DLC(G)=(σ_{DL},μ_{DL}) Where

σ_{DLC}={0.8,0.6, 0.4,0.2}

Here,We need minimum 10colors to proper color all the vertices of the graph DLC($G_{0.2}$).so the chromatic number of DLC ($G_{0.2}$) is 10

For α= 0.2 ,$\chi_{0.2}$ = χ_{DL}(0.2) = 10

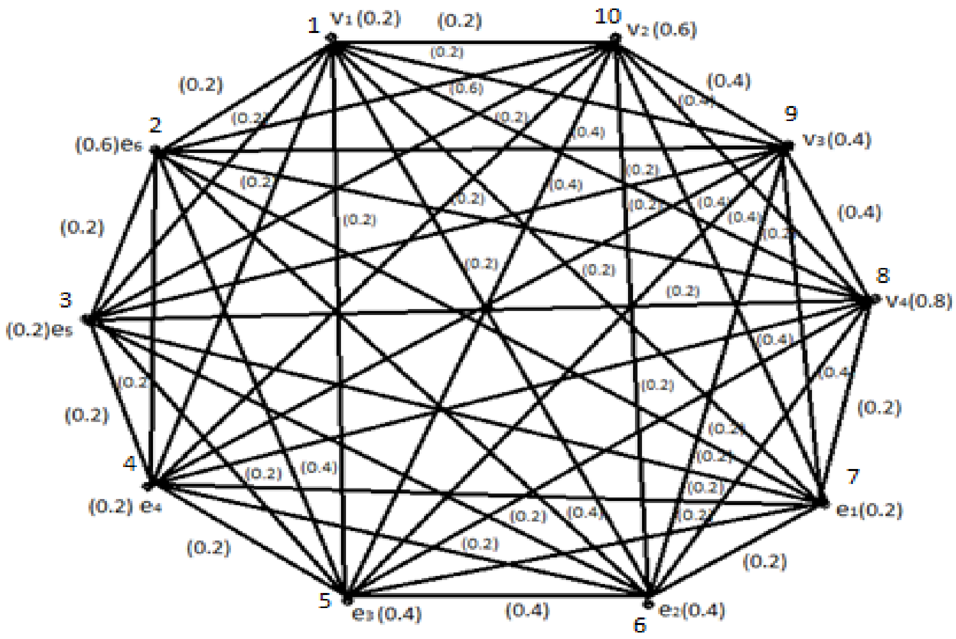


Figure 15

For **α=0.4** Double layered complete fuzzy graph DLC(G)=(σ_{DL},μ_{DL}) Where

σ_{DLC}={0.4}

Here,We need minimum 4 colors to proper color all the vertices of the graph DLC($G_{0.4}$).so the chromatic number of DLC ($G_{0.4}$) is 4

For α= 0.4 ,$\chi_{0.4}$ = χ_{DL}(0.4) = 4

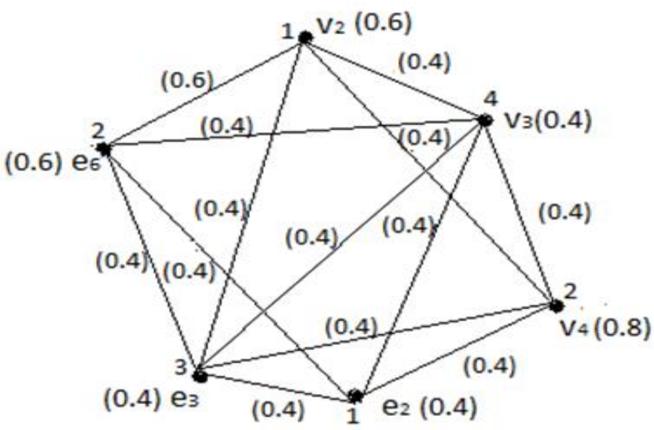


Figure 16

For **α=0.6** Double layered complete fuzzy graph DLC(G)=(σ_{DL},μ_{DL}) Where

σ_{DLC}={0.6}Here,We need minimum 2 colors to proper color all the vertices of the graph DLC($G_{0.6}$).so the chromatic number of DLC ($G_{0.6}$) is 2

For α= 0.6 ,$\chi_{0.6}$ = χ_{DL}(0.6) = 2

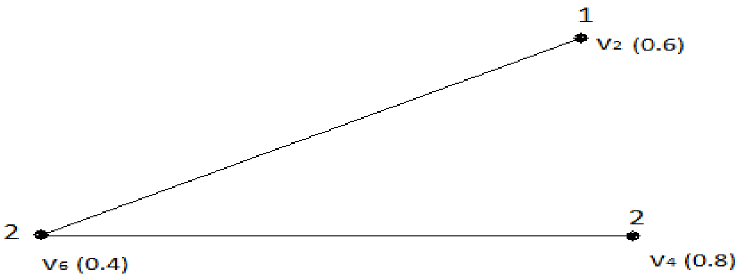


Figure 17

For **α=0.8**Double layered complete fuzzy graph DLC(G)=(σ_{DL},μ_{DL}) Where

σ_{DLC}={0.8}

Here We need minimum 1 colors to proper color all the vertices of the graph DLC($G_{0.8}$).so the chromatic number of DLC ($G_{0.8}$) is 1

For α= 0.8 ,$\chi_{0.8}$ = χ_{DL}(0.8) = 1

Figure 18

THEORITICAL CONCEPT

THEOREM:

The order of Double layered Complete fuzzy graph $\lambda_{\alpha\cup\beta}$ is equal to the sum of the order and size of the complete graph

Proof:

Let $\alpha \cup \beta$ be a complete Double layered fuzzy graph node set and the fuzzy subsetσ_{DL}on and using double layered fuzzy graph,

$\alpha^* \cup \beta^*$ is defined as follows,

$$\sigma_{DL} = \begin{cases} \alpha(\mathrm{u}) \text{ if } \mathrm{u} \in \alpha^* \\ \beta(\mathrm{uv}) \text{ if } \mathrm{uv} \in \beta^* \end{cases}$$

By the definition,order of the double layered fuzzy graph is,

$$O(DL(G)) = \sum_{u\epsilon\sigma\cup\beta} \sigma_{DL}(u)$$

$$= \sum_{u\epsilon\sigma} \sigma_{DL}(u) + \sum_{u\epsilon\beta} \sigma_{DL}(u)$$

$$= \sum_{u\epsilon\sigma} \alpha(u) + \sum_{u\epsilon\beta} \beta(u)$$

O(DL(G)) = Order(G) + Size(G).

3.CONCLUSION

In this paper we can briefly discussed about the coloring of DFCFG using alpha cut. We conclude that the chromatic number is decrease when the value of alpha cut is increase. This concept will help not only in vertex coloring also in edge coloring of DFCFG using alpha cut.

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