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THREE- DIMENSIONAL SCHRODINGER EQUATION AND ITS APPLICATION

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ABSTRACT

The schrodinger equation and extended tanh function in three- dimension are the topics of this study. The major goal of this study is to use the extended tanh function to convert the partial differential Schrodinger equation to ordinary differential Schrodinger equation and its application.

KEYWORDS: Ordinary differential equation, Partial differential equation, Schrodinger equation.

1.INTRODUCTION

The Schrodinger equation is the partial differential equation inn quantum physics that governs the wave function. The Schrodinger equation is often reffered to as the Schrodinger wave equation.It is the most important result in quantum mechanics and its discovery marked a watershed moment in the evolution of the Schrodinger equation. Erwin Schrodinger,an Austrian physicist, devised the Schrodinger in 1925 and published it in 1926,making a significant contribution to quantum mechanics. And laying the groundwork for his Nobel prize winning Physics in 1933.

Despite the fact it has been researched for almost 40 years, the nonlinear Schrodinger equation can be investigated with tremendous interest. The non linear Schrodinger equation in one dimension has the following form

$iq_t+q_{xx}+2|q|^2q$ (1.1)

This equation forms a prospective platform to examine many phenomena in many areas of physics, such as nonlinear optics, because of the simple non linearity and its leadership position in numerous fields. Physics of plasmosphere , superconductivity, quantum physics, Bose-Einstein condensate and nonlinear dynamics of the spin system are the branch of the mathematics that deals with. The major goal of this study is to use modified extended tanh-function to exact solution of damped nonlinear Schrodinger as well as to investigate the effect of nonlocal terms in the single solutions.The significance of the non-locality in wave propagation is a significant component in many physical contests, such as propagation of the cold atomic means in the presence of optically induced dipole-dipole interaction non locally interaction of the ultra -cold atoms in atomic Bose-Einstein condensation, light propagation in the photo- related substance is non -linear and there is no localization. In the non -linear schrodinger equation the electromagnetic wave that regulates three -dimensional solitary wave has the form.

$iq_t+(q_{xx}+q_{yy}+q_{zz})+2|q|^2q$

For solving nonlinear partial equation, equation the tanh technique is a straight forward algebraic method. The Schrodinger Three- Dimensional equation is,

$$iq_t+(1-\lambda)(q_{xx}+q_{yy}+q_{zz})+2|q|^2q+2i\lambda q\int_{-\infty}^{x}\left(qq*_{x'y'z'}-q*q_{x'y'z'}\right)dx'dy'dz'=0$$

Where λ is the damping parameter, schrodinger equation is one of the wave equation.There are two types of schrodinger equation one is time dependent equation and another is time independent schrodingerequation . For a given partial differential equation in three variables

$H(u,u_t,u_x,u_y,u_z,u_{xx}\dots\dots\dots)=0$

2.PRELIMINARIES:

Definition:Schrodinger

The schrodinger equation is building block of quantum mechanics. The position of a partical is distributed through space like the amplitude of a wave.In quantum mechanics , a wave describe the motion and location of a particle. A wave function is just a mathematical function which may be large in one region , small in others and zero elsewhere.

Definition: Modified extended tanh method

The modified extended tanh method is one of the most effective algebraic method for obtaining exact solutions of non- linear partial differential equation.

Definition: Ordinary Differential Equation

An ordinary differential equation contains of an unknown function. The ordinary differential equation is an equation having variable and derivative of the dependent variable with reference to the independent variable.

The two types of ordinary differential equation are homogeneous differential equation and non- homogeneous differential equation. The general form ODE is given by

$$F\left(\frac{dy}{dt}, y, t\right) = 0$$

Definition: Partial Differential Equation

A partial differential equation is an equation involving two or more independent variables. Also with an unknown function and partial derivative of the unknown function with respect to the independent variables.

A partial differential equation partial derivatives of one or more dependent variable with more independent variables. We can shown PDE for the function $U(x_1, x_2, x_3, \ldots\ldots x_n)$ in the form :

$$f\left(x_1 x_2, x_3, \ldots\ldots u, \frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \ldots \frac{\partial^2 u}{\partial^2 x_1}, \ldots\ldots\right) = 0$$

And the PDE will be linear if f is a linear function of u and its derivative can be written the simple PDE as $\frac{\partial u}{\partial x}(x,y)=0$

3.PROCEDURE TO CONVERT PARTIAL DIFFERENTIAL EQUATION TO ORDINARY DIFFERENTIAL EQUATION USING THREE-DIMENSIONAL SCHRODINGER EQUATION.

Step:1 Consider the non- linear schrodinger three- dimensional partial differential equation with non -local damping

Step:2 Using the extended tanh function, the schrodinger three- dimension equation can be calculated.

Step:3 Applying the complex conjugate for the extended tanh function and then separating it by the differentiation.

Step:4 Finally,we acquire three dimensional schrodinger ordinary differential equation by splitting real and imaginary parts of the three -dimensional schrodinger partial differential equation.

3.1 CONVERTING PARTIAL DIFFERENTIAL EQUATION TO ORDINARY DIFFERENTIAL EQUATION.

$iq_t+(1-i\lambda)(q_{xx}+q_{yy}+q_{zz})+2|q|^2q$

Non -linear schrodinger equation with non -local damping with expressed as,

$$iq_t + (1 - i\lambda)(q_{xx} + q_{yy} + q_{zz}) + 2|q|^2 q + 2i\lambda q \int_{-\infty}^{x} (q q^*_{x'y'z'} - q^* q_{x'y'z'}) dx' dy' dz' = 0$$

In non- local damped non- linear schrodinger equation using modified extended tanh function method,

$$R= EG + i\lambda \int_{-\infty}^{x} (EG_{x'y'z'} - GE_{x'y'z'}) dx' dy' dz' \ (4.2a)$$

Following the wave solution

$$R-EG = i\lambda \int_{-\infty}^{x} (EG_{x'y'z'} - GE_{x'y'z'}) dx' dy' dz'$$

And $q= E$, $q *= G$

$$iE_t + (1 - i\lambda)(E_{xx} + E_{yy} + E_{zz}) + 2E^2G+2i\lambda E \int_{-\infty}^{x} (EG_{x'y'z'} - GE_{x'y'z'}) dx' dy' dz'$$

$$iE_t + (1 - i\lambda)(E_{xx} + E_{yy} + E_{zz}) + 2E^2G+2E(R - EG) = 0$$

$iE_t + (1 - i\lambda)(E_{xx} + E_{yy} + E_{zz}) + 2ER = 0$

Now applying complex conjugate for equation (4.2),

$$-iq_t^* + (1 - i\lambda)(q_{xx}^* + q_{yy}^* + q_{zz}^*) + 2|q^*|q^* - 2i\lambda q^* \int_{-\infty}^{x} (q^* q_{x'y'z'} - q q_{x'y'z'}^*) dx' dy' dz' = 0$$

$$-iG_t + (1 + \lambda)(G_{xx} + G_{yy} + G_{zz}) + 2G^2 E + 2GR$$

Differentiate these $(4.2a)$ with respect to x,y,z

$$R_{x'y'z'} - [EG_{x'y'z'} + GE_{x'y'z'}] = i\lambda[EG_{x'y'z'} - GE_{x'y'z'}]$$

$$R_{x'y'z'} - EG_{x'y'z} - GE_{x'y'z'} - i\lambda EG_{x'y'z'} + i\lambda EG_{x'y'z'} = 0$$

$$R_{x'y'z'} - GE_{x'y'z'} + i\lambda GE_{x'y'z'} - EG_{x'y'z'} - i\lambda EG_{x'y'z'} = 0$$

Let E$= u(\varepsilon) e^{i(kx-\omega t)} e^{i(ky-\omega t)} e^{i(kz-\omega t)}$

$\varepsilon = t - \eta(x + y + z) + \varepsilon_0$

R$= V(\varepsilon)$

Therefore, $iE_t = i^4(\omega^3) u(\varepsilon) e^{i(kx-\omega t)} e^{i(ky-\omega t)} e^{i(kz-\omega t)} + iu(\varepsilon) e^{i(kx-\omega t)} e^{i(ky-\omega t)} e^{i(kz-\omega t)}$

$$iE_t = \omega^3 u(\varepsilon) e^{i(kx-\omega t)} e^{i(ky-\omega t)} e^{i(kz-\omega t)} + iu(\varepsilon) e^{i(kx-\omega t)} e^{i(ky-\omega t)} e^{i(kz-\omega t)}$$

2ER$= 2[u(\varepsilon) e^{i(kx-\omega t)} e^{i(ky-\omega t)} e^{i(kz-\omega t)}].[v(\varepsilon)]$

$$E_{xx} - i\lambda E_{xx} = [\eta^2 u(\varepsilon)'' e^{i(kx-\omega t)} - 2\eta u(\varepsilon)' ike^{i(kx-\omega t)} - u(\varepsilon) k^2 e^{i(kx-\omega t)} + i\lambda k^2 u(\varepsilon) e^{i(kx-\omega t)} - 2\eta\lambda ku(\varepsilon)' e^{i(kx-\omega t)} - i\omega\eta^2 u(\varepsilon)'' e^{i(kx-\omega t)}] e^{i(ky-\omega t)} e^{i(kz-\omega t)}$$

$$E_{yy} - i\lambda E_{yy} = [\eta^2 u(\varepsilon)'' e^{i(ky-\omega t)} - 2\eta u(\varepsilon)' ike^{i(ky-\omega t)} - u(\varepsilon) k^2 e^{i(ky-\omega t)} + i\lambda k^2 u(\varepsilon) e^{i(ky-\omega t)} - 2\eta\lambda ku(\varepsilon)' e^{i(ky-\omega t)} - i\omega\eta^2 u(\varepsilon)'' e^{i(ky-\omega t)}] e^{i(kx-\omega t)} e^{i(kz-\omega t)}$$

$$E_{zz} - i\lambda E_{zz} = [\eta^2 u(\varepsilon)'' e^{i(kz-\omega t)} - 2\eta u(\varepsilon)' ike^{i(kz-\omega t)} - u(\varepsilon) k^2 e^{i(kz-\omega t)} + i\lambda k^2 u(\varepsilon) e^{i(kz-\omega t)} - 2\eta\lambda ku(\varepsilon)' e^{i(kz-\omega t)} - i\omega\eta^2 u(\varepsilon)'' e^{i(kz-\omega t)}] e^{i(kx-\omega t)} e^{i(ky-\omega t)}$$

Substitute above value in (4.6)

$$[\omega^3 u(\varepsilon) + iu(\varepsilon)' + 3\eta^2 u(\varepsilon)'' + 6\eta u(\varepsilon) ik - 3u(\varepsilon) k^2 + 3ik^2 u(\varepsilon) - 6\eta\lambda u(\varepsilon)'' + 3i\lambda\eta^2 u(\varepsilon)'' + 2u(\varepsilon) v(\varepsilon)] = 0.$$

For i=-i

$$[\omega^3 u(\varepsilon) - iu(\varepsilon)' + 3\eta^2 u(\varepsilon)'' - 6\eta u(\varepsilon) ik - 3u(\varepsilon) k^2 - 3ik^2 u(\varepsilon) - 6\eta\lambda u(\varepsilon)'' - 3i\lambda\eta^2 u(\varepsilon)'' + 2u(\varepsilon) v(\varepsilon)] = 0.$$

By using

$$R_{x'y'z'} + i\lambda GE_{x'y'z'} - GE_{x'y'z'} - i\lambda EG_{x'y'z'} - EG_{x'y'z'} = 0$$

$$R_{x'y'z'} = (-\eta) + (-\eta) + (-\eta)$$

$$GE_{x'y'z'} = 3u(\varepsilon)^2 ik - 3\eta u(\varepsilon) u(\varepsilon)'$$

$$i\lambda GE_{x'y'z'} = -\lambda 3u(\varepsilon)^2 k - 3i\lambda\eta u(\varepsilon) u(\varepsilon)'$$

$$EG_{x'y'z'} = -3u(\varepsilon)^2 ik - 3\eta u(\varepsilon) u'$$

$$-i\lambda EG_{x'y'z'} = -3\lambda u(\varepsilon)^2 k + i\lambda 3\eta u(\varepsilon)u(\varepsilon)'$$

Substitute these value in

$$-3\eta V(\varepsilon)' - 6\lambda u(\varepsilon)^2 + 6\eta u(\varepsilon)u(\varepsilon)' = 0$$

By separating real and imaginary parts ,we obtain

REAL PART :

$$\omega^3 u(\varepsilon) + 3\eta^2 u(\varepsilon)'' - 3u(\varepsilon)k^2 - 6\eta k u(\varepsilon)'' + 2u(\varepsilon)v(\varepsilon)$$

IMAGINARY PART:

$$u(\varepsilon)' + 6\eta u(\varepsilon) + 3k^2 u(\varepsilon) - 3\lambda\eta^2 u(\varepsilon)''$$

Finally we conclude that

$$\omega^3 u(\varepsilon) + 3\eta^2 u(\varepsilon)'' - 3u(\varepsilon)k^2 - 6\eta k u(\varepsilon)'' + 2u(\varepsilon)v(\varepsilon) - 3\eta v(\varepsilon)' - 6\lambda u(\varepsilon)^2 k + 6\eta u(\varepsilon)u(\varepsilon)'$$

3.2 APPLICATION OF THREE-DIMENSIONAL SCHRODINGER EQUATION.

The time independent Schrodinger equation is used for a number of practical problem. Systems with bound states are related to the quantum mechanics particle in a box, barrier penetration is important in radioactive decay, and the quantum mechanical oscillator is applicable to the molecular vibrational modes.

The particle in a box problem is a common application of quantum mechanical model to simplified system consisting of a particle moving horizontally within an infinitely deep well from which it cannot escape.

The solutions to the problem give possible values of E and ψ that particle can possess.

E represents allowed energy values and $\psi(x)$is a wave function, which when squared gives us the probability of locating the particle at a certain position within the box at given energy level.

To solve the problem for a particle in a 1-dimensional box, we must follow the recipe for quantum mechanics:One dimensional Schrodinger Equation define the Potential Energy, we solve the Schrodinger Equation define the wave-functions solved for allowed energies.

All of the information for a subatomic particle is encoded within a wave function. The wave function will satisfy and can be solved by using the Schrodinger equation. The Schrodinger equation is one of the fundamental axioms that are introduced in undergraduate physics. It is also increasing common to find the Schrodinger equation being introduced within the electrical engineering syllabus in universities as it is applicable to semiconductor.

The problem can be solved by expressing the differential Schrodinger equation in momentum space \reciprocal space, where the Schrodinger equation can be expressed as a set of linear algebraic equations rather than complex differential equations.

Today it is possible to obtain a suitable dopant for a given semiconductor just by solving the corresponding Schrodinger equation, without conducting any experiments.

The material in practice are impure, they contain various impurities as well as crystalline defects. The ground state properties of these defects can be well studied by solving the Schrodinger equation.

4. CONCLUSION

The concept of the Schrodinger equation been briefly examined. For the non-linear partial differential equation, we obtain the numerous travelling wave solution. The extended tanh function method's validity and reliability are assessed by applying it to a non-linear partial differential equation. Using the tanh function approach and the hyperbolic function method, the acquired findings are compared to non-local damping. A new solution is exact. It will be used to solve the logarithmic schrodinger equation in the future.

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