

OCTAGONAL NEUTROSOPHIC NUMBER AND THEIR APPLICATION IN TRANSPORTATION PROBLEM ENVIRONMENT USING RUSSELL'S APPROXIMATION METHOD

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ABSTRACT

In this paper, dealt with the Octagonal Neutrosophic number sets and its operational law . It contains the single valued Octagonal Neutrosophic number (ONN) and in addition to that, the arithmetic operation of single valued ONN has been discussed. By the Russell's Approximation method (RAM) , the transportation problem with ONN has been carried out. In this method the minimum cost of transportation problem has been analyzed.

Keywords: Octagonal number, Single-valued Octagonal Neutrosophic numbers, Accuracy function, Neutrosophic numbers.

INTRODUCTION:

Hitchcock pioneered the basic transportation problem. This type of traditional problem can be as a direct programming problem and subsequently addressed the simplex strategy. This type of classical problem can be modeled as a linear programming problem and then solved simplex method. A primary simplex method to the transportation problem was solved by Dantzig and Thapa. The Transportation Problem is a different type of structure , so simplex method is not suitable for finding the objectives. Due to some drawback in simplex method for solving TP, a new Initial Basic Feasible Solution (IBFS) method was developed. By using the IBFS, there are three type of methods (1) north-west corner (NWC), (2) least-cost method, (3) vogel's approximation method. In classic TP the decision makers knows the values of supply, demand and transportation cost i.e. the decision makers consider the crisp numbers. However, in our day to day applications, the decision makers may not be known precisely to all the parameters of transportation problem due to some uncontrolled factor. To overcome this uncontrolled factor, fuzzy decision making method is introduced.

The basic concepts of fuzzy set theory was introduced by Zadeh in 1965. Since then , several researchers have studied the fuzzy transportation problem (FTP). A Fuzzy Linear Programming (FLP) problem was proposed by Zimmerman and he has shown that the method was always very efficient.

Smarandache introduced in 1988, the structure of Neutrosophic Set (NS) which is a superior version of both fuzzy and the intuitionistic fuzzy. Neutrosophic set could be described by three autonomous degrees, namely

- (i) Truth-membership degree (T),
- (ii) Indeterminacy membership degree (I),
- (iii) Falsity membership degree (F).

Subsequently, Wang et al. introduced a Single Value Neutrosophic Set (SVNS) problem to solve a practical problem. Ye introduced the Trapezoidal Neutrosophic Set (TrNS) by combining the concept of Trapezoidal Fuzzy Numbers (TrFN) and SVNS . To exploit the beauty advantages of NS, several researchers have proposed a different method to solve LP problem under Neutrosophic environment. Das and Dash proposed a modified solution of LP Neutrosophic problem. Recently, Das and Chakraborty proposed a new approach to solve LP problem in pentagonal Neutrosophic environment.

2. PRELIMINARIES:

Definition 2.1: Fuzzy sets

A set \tilde{A} is denoted as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0, 1]\}$ represented by $(x, \mu_{\tilde{A}}(x))$, where $x \in$ the crisp set X and $\mu_{\tilde{A}}(x) \in$ the interval $[0, 1]$, the set \tilde{A} is called fuzzy set.

Definition 2.2: Neutrosophic Fuzzy Number

Let U be a universe of discourse then the Neutrosophic set A is an object having the form $A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle; x \in U \}$ where the functions $T, I, F : U \rightarrow [0, 1]$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3: Score Function

Let us consider a single valued Octagonal Neutrosophic Numbers (ONN) as $\tilde{F}_{oct} = (F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8; \pi, \sigma, \rho)$, The primary application of score function is to drag the judgment of conversion of ONN to crisp number. Also, the mean of the PNN components is $(F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8)$ and score value of the membership portion is $\{2 + \pi - \rho - \sigma\}$.

Thus, for a O.N.N $\tilde{F}_{oct} = (F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8; \pi, \sigma, \rho)$. Score function is scaled as $S\tilde{C}_{oct} = \frac{1}{24} (F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8) \times \{2 + \pi - \rho - \sigma\}$, $S\tilde{C}_{oct} \in \mathbb{R}$.

Definition 2.4: Pentagonal Neutrosophic Number

Pentagonal Neutrosophic Number PNN is defined as,

$$PNN = \langle [(a, b, c, d, e); \theta], [(a^1, b^1, c^1, d^1, e^1); \Psi], [(a^2, b^2, c^2, d^2, e^2); \mu] \rangle$$

Where $\theta, \Psi, \mu \in [0, 1]$.

The truth membership function (θ): $\mathbb{R} \rightarrow [0, \theta]$,

The indeterminacy membership function (Ψ): $\mathbb{R} \rightarrow [\mu, 1]$,

The falsity membership function (μ): $\mathbb{R} \rightarrow [\Psi, 1]$

Definition 2.5 : Single valued Octagonal Neutrosophic Number [SONN]

A Octagonal Neutrosophic Number denoted by,

$$\tilde{S} = \langle [(a, b, c, d, e, f, g, h); \theta], [(a^1, b^1, c^1, d^1, e^1, f^1, g^1, h^1); \Psi], [(a^2, b^2, c^2, d^2, e^2, f^2, g^2, h^2); \mu] \rangle$$

Where $\theta, \Psi, \mu \in [0, 1]$.

The truth membership function (θ_s): $\mathbb{R} \rightarrow [0, \theta]$,

The indeterminacy membership function (Ψ_s): $\mathbb{R} \rightarrow [\mu, 1]$,

The falsity membership function (μ_s): $\mathbb{R} \rightarrow [\Psi, 1]$

Condition 1:

1. θ_s : Truth membership function (θ_s): $\mathbb{R} \rightarrow [0, \theta]$,

2. Ψ_s : Indeterminacy membership function (Ψ_s): $\mathbb{R} \rightarrow [\mu, 1]$,

3. μ_s : Falsity membership function (μ_s): $\mathbb{R} \rightarrow [\Psi, 1]$.

Condition 2:

1. $\theta_{\mathcal{S}}$: Truth membership function is strictly non-decreasing continuous function on the intervals [a, e].
2. $\Psi_{\mathcal{S}}$: Indeterminacy membership function is strictly non-decreasing continuous function on the intervals [a¹, e¹].
3. $\mu_{\mathcal{S}}$: Falsity membership function is strictly non-decreasing continuous function on the intervals [a², e²].

Condition 3:

1. $\theta_{\mathcal{S}}$: Truth membership function is strictly non-increasing continuous function on the intervals [e, h].
2. $\Psi_{\mathcal{S}}$: Indeterminacy membership function is strictly non-increasing continuous function on the intervals [e¹, h¹].
3. $\mu_{\mathcal{S}}$: Falsity membership function is strictly non-increasing continuous function on the intervals [e², h²].

3. PROPERTIES OF OCTAGONAL NEUTROSOPHIC NUMBER**3.1 Relationship between any two Octagonal Neutrosophic number:**

Let us consider the any two Octagonal Neutrosophic fuzzy number defined as follows, $\tilde{S}_{OctNeu1}=(T_{OctNeu1}, I_{OctNeu1}, F_{OctNeu1})$ and $\tilde{S}_{OctNeu2}=(T_{OctNeu2}, I_{OctNeu2}, F_{OctNeu2})$ if,

1. $SC_{OctNeu1} > SC_{OctNeu2}$ then $\tilde{S}_{OctNeu1} > \tilde{S}_{OctNeu2}$
2. $SC_{OctNeu1} < SC_{OctNeu2}$ then $\tilde{S}_{OctNeu1} < \tilde{S}_{OctNeu2}$
3. $SC_{OctNeu1} = SC_{OctNeu2}$ then $\tilde{S}_{OctNeu1} = \tilde{S}_{OctNeu2}$
4. $AC_{OctNeu1} > AC_{OctNeu2}$ then $\tilde{S}_{OctNeu1} > \tilde{S}_{OctNeu2}$
5. $AC_{OctNeu1} < AC_{OctNeu2}$ then $\tilde{S}_{OctNeu1} < \tilde{S}_{OctNeu2}$
6. $AC_{OctNeu1} = AC_{OctNeu2}$ then $\tilde{S}_{OctNeu1} = \tilde{S}_{OctNeu2}$

3.2 ARITHMETIC OPERATION:

Let $\tilde{S}_1 = \langle (p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8) \pi_{\delta_1}, \mu_{\delta_1}, \rho_{\delta_1} \rangle$ and $\tilde{S}_2 = \langle (q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8) \pi_{\delta_2}, \mu_{\delta_2}, \rho_{\delta_2} \rangle$ be two Octagonal Neutrosophic Number and $\alpha \geq 0$. Then the following operational relations holds:

$$\tilde{S}_1 + \tilde{S}_2 = \langle (p_1+q_1, p_2+q_2, p_3+q_3, p_4+q_4, p_5+q_5, p_6+q_6, p_7+q_7, p_8+q_8); \max\{\pi_{\delta_1}, \pi_{\delta_2}\}, \min\{\mu_{\delta_1}, \mu_{\delta_2}\}, \min\{\rho_{\delta_1}, \rho_{\delta_2}\} \rangle$$

$$\tilde{S}_1 - \tilde{S}_2 = \langle (p_1 - q_8, p_2 - q_7, p_3 - q_6, p_4 - q_5, p_5 - q_4, p_6 - q_3, p_7 - q_2, p_8 - q_1); \max\{\pi_{\delta_1}, \pi_{\delta_2}\}, \min\{\mu_{\delta_1}, \mu_{\delta_2}\}, \min\{\rho_{\delta_1}, \rho_{\delta_2}\} \rangle$$

$$\alpha \tilde{S}_1 = \langle (\alpha p_1, \alpha p_2, \alpha p_3, \alpha p_4, \alpha p_5, \alpha p_6, \alpha p_7, \alpha p_8); 1 - (1 - \pi_{\delta_1})^\alpha, \mu_{\delta_1}^\alpha, \rho_{\delta_1}^\alpha \rangle$$

$$\tilde{S}_1^\alpha = \langle (p_1^\alpha, p_2^\alpha, p_3^\alpha, p_4^\alpha, p_5^\alpha, p_6^\alpha, p_7^\alpha, p_8^\alpha); \pi_{\delta_1}^\alpha, (1 - \mu_{\delta_1})^\alpha, (1 - \rho_{\delta_1})^\alpha \rangle.$$

4. PROPOSED METHOD:**ALGORITHM FOR RUSSELL'S APPROXIMATION METHOD:**

Step:1 First convert the octagonal number given in the table as score function.

Step:2 Check whether the problem is Balanced or Unbalanced [i.e., If Demand=supply do not add dummy row or column to make it balanced problem. If Demand≠Supply we should introduce the dummy row or column].

Step:3 For each supply row determine the largest cost of the row(A_i).

Step:4 For each supply column determine the largest cost of the column(B_j).

Step:5 For each variable calculate $\theta_{ij}=C_{ij}-(A_i+B_j)$.

Step:6 Select the variable having the most negative θ value and the minimum supply or demand and made the allotment in the cell having the least unit cost.

Step:7 Again return to step 3 and repeat the process as much as possible and eliminate the necessary cell consider.

Step:8 Substitute all the allotted values with the corresponding variables and we get the transportation cost.

5.Numerical Example:

EXAMPLE: In Geneva, Switzerland have a company named JOHN AUTOMOBILES Pvt,Ltd. And the organization has 3 plants for delivering Automobile spare parts. The spare parts ought to be transport to 3 places under Octagonal Neutrosophic numbers. The conditions of transportation problem are presented in Table 1 . As the problem should be ONN therefore , the decision-maker consider the confirmation degree of Octagonal number is (1,0,0).

Table:1

Factory	Meyrin	Lancy	Versoix	Supply
Kelvin	(5,10,13,14,18,20,22,24;1,0,0)	(1,2,3,4,5,6,7,8;1,0,0)	(2,6,8,14,14,16,18,20;1,0,0)	(2,11,23,34,45,56,67,78;1,0,0)
Peter	(3,4,5,6,7,8,9,10;1,0,0)	(1,5,6,7,11,12,13,17;1,0,0)	(1,4,5,9,16,17,19,21;1,0,0)	(10,47,52,65,76,80,84,85;1,0,0)
David	(3,6,9,12,15,18,21,24;1,0,0)	(2,5,7,8,8,9,9,10;1,0,0)	(1,1,1,1,1,1,1,1;1,0,0)	(3,18,56,76,87,90,92,95;1,0,0)
Demand	(11,16,51,67,75,79,81,83;1,0,0)	(20,40,60,80,100,120,140,160;1,0,0)	(15,30,45,75,110,115,120,125;1,0,0)	

Solution:

Step:1 First convert the Octagonal Neutrosophic number given in the table as Score function.

Factory	Meyrin	Lancy	Versoix	Supply
Kelvin	15.75	4.5	12.25	39.5
Peter	6.5	9	11.5	62.375
David	13.5	7.25	1	64.62
Demand	57.875	90	79.375	

Step:2 Check whether the problem is balanced or unbalanced.

(i.e,Supply = Demand)

Supply=39.5+62.375+64.62

Demand=57.875+90+79.375

Supply=166.495

Demand=227.25

Hence,Supply \neq Demand . So we add Dummy row in order to make this problem balanced one.

Factory	Meyrin	Lancy	Versoix	Supply
Kelvin	15.75	4.5	12.25	39.5
Peter	6.5	9	11.5	62.375
David	13.5	7.25	1	64.62
Dummy	0	0	0	60.755
Demand	57.875	90	79.375	

Step:3 For each supply row determine the largest cost of row(A_i) and demand column determine the largest cost of column(B_j).

➤ Compute the reduced cost to each ∂_{ij} where $\partial_{ij}=C_{ij}-(A_i+B_j)$

Factory	Meyrin	Lancy	Versoix	Supply	A_i
Kelvin	15.75	4.5	12.25	39.5	15.75
Peter	6.5	9	11.5	62.375	11.5
David	13.5	7.25	1	64.62	13.5
Dummy	0	0	0	60.755	0
Demand	57.875	90	79.375		
B_j	15.75	9	12.25		

$$\partial_{11}=C_{11}-(A_1+B_1)=15.75-(15.75+15.75)= -15.75$$

$$\partial_{12}=C_{12}-(A_1+B_2)=4.5-(15.75+9)= -20.25$$

$$\partial_{13}=C_{13}-(A_1+B_3)=12.25-(15.75+12.25)= -15.75$$

$$\partial_{21}=C_{21}-(A_2+B_1)=6.5-(11.5+15.75)= -20.75$$

$$\partial_{22}=C_{22}-(A_2+B_2)=9-(11.5+9)= -11.5$$

$$\partial_{23}=C_{23}-(A_2+B_3)=11.5-(11.5+12.25)= -12.25$$

$$\partial_{31}=C_{31}-(A_3+B_1)=13.5-(13.5+15.75)= -15.75$$

$$\partial_{32}=C_{32}-(A_3+B_2)=7.25-(13.5+9)= -15.25$$

$$\partial_{33}=C_{33}-(A_3+B_3)=1-(13.5+12.25)= -24.75$$

$$\partial_{41}=C_{41}-(A_4+B_1)=0-(0+15.75)= -15.75$$

$$\partial_{42}=C_{42}-(A_4+B_2)=0-(0+9)= -9$$

$$\partial_{43}=C_{43}-(A_4+B_3)=0-(0+12.25)= -12.25$$

Step:4 Select the variable having the most negative ∂ value and select the minimum supply or demand and made the allotment in the cell having the least unit cost.

Factory	Meyrin	Lancy	Versoix	Supply	A_i
Kelvin	15.75 [-15.7]	4.5 [-20.25]	12.25 [-15.75]	39.5	15.75
Peter	6.5 [-20.75]	9 [-11.5]	11.5 [-12.25]	62.375	11.5
David	13.5 [-15.75]	7.25 [-15.25]	1 [-24.75] [64.62]	64.62	13.5
Dummy	0 [-15.75]	0 [-9]	0 [-12.25]	60.755	0
Demand	57.875	90	79.375 [79.375- 64.62=14.755]		
B_j	15.75	9	12.25		

The most negative value is $\partial_{33}= -12.75$

$$\text{Min}(64.62,79.375)=64.62$$

Step:5

Return step:3 and repeat the process as much as possible and eliminate the necessary cell consider.

Step:6

Factory	Meyrin	Lancy	Versoix
Kelvin	15.75	4.5 [39.5]	12.25
Peter	6.5 [57.875]	9 [4.5]	11.5
David	13.5	7.25	1 [64.62]
Dummy	0	0 [46]	0 [14.755]

The minimum Transportation cost is obtained as:

$$\text{Min}=(4.5 \times 39.5)+(6.5 \times 57.875)+(9 \times 4.5)+(1 \times 64.62)+(0 \times 4.5)+(0 \times 14.755)$$

=659.0575

Here, The number of allocated cells is 6.

Which is equal to $m+n-1=4+3-1=6$.

Therefore, this solution is non-degenerate.

Conclusion:

Operation research, The transportation problem is the most preferred one among other updation problems. The main objective of this transportation problem is finding the minimum cost of transportation between supplier and demand. In this paper, we have solved the Neutrosophic transportation problem under Octagonal Neutrosophic numbers (ONN). When dealing with the analytical point of view, our method is the most precise and less time consuming in our day to day life situation. Furthermore this method also extend in application of Octagonal assignment problem, Octagonal linear fractional programming and Octagonal job scheduling problem.

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