ISSN: 0974-5823 Vol. 7 No. 4 April, 2022

International Journal of Mechanical Engineering

SOLVING PYTHAGOREAN TRANSPORTATION PROBLEM USING ARITHMETIC MEAN AND HARMONIC MEAN

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ABSTRACT

In this paper, Arithmetic and Harmonic mean is introduced to resolve a Pythagorean Fuzzy Transportation Problem (PFTP) and an algorithm is presented for the suggested method. Numerical examples are illustrated and the end result is compared with other existing methods and the method which we introduced here gives the better optimum solutions.

KEYWORDS: Arithmetic mean, Harmonic mean, Accuracy function, Score function, Pythagorean Fuzzy Sets (PFS)

1. INTRODUCTION:

Transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. The main aim of the transportation problem is to transfer goods from one place to another place with minimum cost and maximum profit. In recent world, all the constraints of the transportation problem may not be known completely due to intractable characteristics, in order to overcome this situation, fuzzy numbers are initiated by Zadeh[10] in 1965 and latter developed by Zimmermann[5] in 1978.Yager[4] established an additional category of non-standards fuzzy subset called Pythagorean Fuzzy Set (PFS) which is a special case used to overcome the situation that if the sum of the membership function and non-membership function is greater than one. In this work, a proposed algorithm using arithmetic and harmonic mean to unravel the PFTP is suggested.

This paper is constructed as below. Definitions for Pythagorean Fuzzy Sets and for arithmetic and harmonic mean are provided in section 2. Mathematical model for PFTP is exhibited in section 3. A modified algorithm for arithmetic and harmonic mean is deliberated in section 4.An numerical examples are illustrated in section 5 and the conclusion is given in section 6.

2. PRELIMINARIES

Definition 1(Pythagorean Fuzzy Set)

Let X is a fixed set, a Pythagorean Fuzzy Set is an object having the form, P={x,$(\theta_p(\mathrm{x}),\delta_p(\mathrm{x}))$|x ∈ X},where the function : X→ [0, 1] and : X→[$\delta_p(\mathrm{x})$:X [0,1]are the degree of membership and non-membership of the element x ∈X to P, respectively. Also for every x ∈ X, it holds that $(\theta_p(\mathrm{x}))^2$+$(\delta_p(\mathrm{x}))^2 \leq 1$.

Definition 2 (Properties)

Let $\breve{a}_1^p$= (θ_i^p,δ_s^p) and $\breve{b}$= (θ_i^p,δ_f^p) be two Pythagorean Fuzzy Numbers (PFNs).Then the arithmetic operations are as follows:

(i)Additive property: $a_1^p \oplus b_1^p$= $\sqrt{(\theta_p^i)^2+(\theta_0^i)^2-(\theta_p^i)^2(\theta_0^i)^2}, (\delta_s^p.\delta_f^p)$

(ii)Multiplicative property: $a_1^p \otimes b_1^p$=$(\theta_i^p.\delta_s^p, \sqrt{(\theta_p^i)^2+(\theta_p^i)^2)^2-(\delta_s^p)^2(\delta_f^p)^2})$)

(iii)Scalar product: $= ka_1^p$= ($\sqrt{1-(1-\theta_i^p)^k(\delta_s^p)^k}$)), where k is nonnegative const ... that is, k>0.

Definition-3(Comparison of two PFNs)

Let $\breve{a}_1^p = (\theta_i^p, \delta_s^p)$ and $b_1^p = (\theta_0^p, \delta_f^p)$ be two Pythagorean Fuzzy Numbers such that the score and accuracy function are as follows:

(i)Score function: S ($\breve{a}_1^p$) = $\frac{1}{2}(1-(\theta_p^i)^2-(\delta_s^p)^2)$ ---------- (1)

(ii)Accuracy function: ($\breve{a}_1^p$) = $(\theta_p^i)^2+(\delta_s^p)^2$ ---------------- (2)

Then the following five cases arise:

Case 1: If $\breve{a}_1^p > b_1^p$ if and only if S ($\breve{a}_1^p$) > S (b_1^p)

Case 2: If $\breve{a}_1^p < b_1^p$ if and only if S ($\breve{a}_1^p$) < S (b_1^p)

Case 3: If S ($\breve{a}_1^p$) = S (b_1^p) and H ($\breve{a}_1^p$) < H (b_1^p), then $\breve{a}_1^p < b_1^p$

Case 4: If S ($\breve{a}_1^p$) = S (b_1^p) and H ($\breve{a}_1^p$) > H (b_1^p), then $\breve{a}_1^p > b_1^p$

Case 5: If S ($\breve{a}_1^p$) = S (b_1^p) and H ($\breve{a}_1^p$) = H (b_1^p), then $\breve{a}_1^p = b_1^p$

Definition-4 (Arithmetic Mean)

The arithmetic mean is the ratio of all observations in a data set. It is the ratio of sum of all observations. In general, arithmetic mean is defined as,

$$A = \frac{1}{n}\sum_{i=1}^{n} a_i \text{--------------} \quad (3)$$

Definition-5 (Harmonic Mean)

The harmonic mean is one of the types of numerical average. It is calculated by dividing the number of observations by the reciprocal of each number in the series.

$$HM = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

3. Model of Pythagorean fuzzy transportation problem

The balanced Pythagorean fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

c_{ij}^p=The Pythagorean fuzzy transportation cost for unit quantity of the product from it i^{th} source to j^{th}destination.

a_{ij}^p= the Pythagorean fuzzy availablity of the product at i^{th} source.

b_{ij}^p= the Pythagorean fuzzy demand of the product at j^{th}destination

x_{ij} = the fuzzy quantity of the product that should be transported from i^{th}source to j^{th} destination to minimize the total fuzzy transportation cost.

Pythagorean fuzzy transportation problem is given by,

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^p * x_i^j$------------------------ (4)

Subject to $\sum_{j=1}^{n} a_i^p, i = 1,2,3,\dots,m$

$$\sum_{i=1}^{m} x\,ij, i = b_{ij}^{p}, j = 1,2,3, \ldots$$

$$\sum_{j=1}^{n} a_i^p = \sum_{j=1}^{n} b_i^p, i = 1,2,3, \ldots, \mathrm{m}$$

4. Algorithm using arithmetic and harmonic mean to solve PFTP

Step-1: Check whether the given problem is balanced or not. If not, introduce dummy row/column to make the balanced.

Step-2: Calculate the arithmetic mean and harmonic mean for each and every row and column using the formulas given above for each corresponding problems.

Step-3: Now choose the maximum arithmetic and harmonic mean values from the corresponding problems from (step-2), and assign the minimum (supply/demand) at the place of lowest value of the consequent row/column.

Step-4: Repeat the step 2 and 3 till the demand and supply are fatigued.

Step-5: Compute the total transportation cost of PFTP.

5. Numerical Example for Arithmetic Mean:

The input data for PFTP is given below. The intent of the proposed method is to minimize the transportation cost and maximize the profit. The same problem used in [6] is taken for verification.

Table 1: Pythagorean fuzzy transportation problem

	D1	D2	D3	D4	Supply
O_1	(0.3,0.6)	(0.4,0.6)	(0.8,0.4)	(0.6,0.4)	25
O_2	(0.4,0.3)	(0.7,0.4)	(0.5,0.7)	(0.7,0.4)	26
O_3	(0.6,0.2)	(0.8,0.2)	(0.7,0.3)	(0.9,0.1)	29
Demand	18	22	27	13	

In the given table, both the supply and demand are equal and has the outcome of 80. Hence the transportation problem is balanced. Here the total supply and total demand are equal. Hence we can proceed to step 2. By the definition 3, the score function is given by,

Score function,

Now we have to convert PFN into CN. Here, we use the score function for converting the Pythagorean fuzzy numbers into crisp numbers.

Applying the score function to all values, we can convert all the Pythagorean fuzzy numbers into crisp numbers. So that, we get the defuzzified PFTP

The defuzified Pythagorean fuzzy transportation problem is given below:

Table 2: Defuzzified Pythagorean fuzzy transportation problem

	D_1	D_2	D_3	D_4	Supply
O_1	0.275	0.24	0.1	0.24	25
O_2	0.375	0.175	0.13	0.175	26
O_3	0.3	0.16	0.21	0.09	29
Demand	18	22	27	13	

Step 2: Find the AM using () for every row and column and write it below the corresponding rows and columns.

Table 3: Arithmetic mean of the cost values

	D_1	D_2	D_3	D_4	Supply	AM
O_1	0.275	0.24	0.1	0.24	25	0.21375
O_2	0.375	0.175	0.13	0.175	26	0.21375
O_3	0.3	0.16	0.21	0.09	29	0.19
Demand	18	22	27	13		
AM	0.3166	0.1916	0.1466	0.1683		

Step 3: Choose the maximum Arithmetic mean values from table 3, and assign the min (supply/demand) at the place of lowest value of consequent row or column. The maximum Arithmetic mean is at the first column and the minimum allocation is at the cell (1, 1).

Table 4: First allocation by AM

	D_1	D_2	D_3	D_4	Supply	AM
O_1	0.275(18)	0.24	0.1	0.24	25(7)	0.21375
O_2	0.375	0.175	0.13	0.175	26	0.21375
O_3	0.3	0.16	0.21	0.09	29	0.19
Demand	18	22	27	13		
AM	0.3166	0.1916	0.1466	0.1683		

Repeating the procedure until all the rim requirements are satisfied.

	D_1	D_2	D_3	D_4	Supply
O_1	0.275(18)	0.24	0.1(7)	0.24	25
O_2	0.375	0.175	0.13(20)	0.175(6)	26
O_3	0.3	0.16(16)	0.21	0.09(3)	29
Demand	18	22	27	13	

The above table satisfies the rim conditions with (m+n-1) non-negative allocations at independent positions.

Thus the optimal allocation is: $x_{11}=18, x_{13}=7, x_{23}=20, x_{24}=6 \; x_{33}=16, x_{34}=13$

The transportation cost is,=(0.275x18)+(0.1x7)+(0.13x20)+(0.175x6)+(0.16x16)+(0.09x13)=4.95+0.7+2.6+1.05+2.56+ 1.17 =13.03

Total minimum cost will be Rs.13.03

The problem which we used here in the method of arithmetic mean gives the minimum cost, when comparing with other existing methods like NWC, LCM, Heuristic Method gives the better optimum results.

Table 6: Comparison table

NWC method	LCM Method	Heuristic Method	Arithmetic Mean Method
15.21	14.29	14.71	13.03

6. Numerical Example for Harmonic Mean

The input data for PFTP is given below. The intent of the proposed method is to minimize the transportation cost and maximize the profit. The same problem used in [6] is taken for verification for the harmonic mean also. The same algorithm and the same process is continue for harmonic mean also. Hence we start from table,

Table 7: Defuzzified Pythagorean fuzzy transportation problem

	D_1	D_2	D_3	D_4	Supply
O_1	0.275	0.24	0.1	0.24	25
O_2	0.375	0.175	0.13	0.175	26
O_3	0.3	0.16	0.21	0.09	29
Demand	18	22	27	13	

Table 8: Harmonic mean for the cost values

	D_1	D_2	D_3	D_4	Supply	HM
O_1	0.275	0.24	0.1	0.24	25	0.1820
O_2	0.375	0.175	0.13	0.175	26	0.1835
O_3	0.3	0.16	0.21	0.09	29	0.1517
Demand	18	22	27	13		
HM	0.3113	0.1859	0.1336	0.1429		

Table 9: First allocation by HM

	D_1	D_2	D_3	D_4	Supply
O_1	0.275(18)	0.24	0.1	0.24	25(7)
O_2	0.375	0.175	0.13	0.175	26
O_3	0.3	0.16	0.21	0.09	29
Demand	18	22	27	13	

Repeating the procedure until all the rim requirements are satisfied

Table 10: Optimal solution of PFTP

	D_1	D_2	D_3	D_4	Supply
O_1	0.275(18)	0.24	0.1(1)	0.24(6)	25(7)
O_2	0.375	0.175	0.13	0.175	26
O_3	0.3	0.16(22)	0.21	0.09(7)	29
Demand	18	22	27	13	

The above table satisfies the rim conditions with (m+n-1) non-negative allocations at an independent positions.

Thus the optimal allocation is: $x_{11}=18, x_{13}=1, x_{23}=26, x_{32}=22, x_{34}=7, x_{14}=7$

The transportation cost is
=(0.275x18)+(0.1x1)+(0.24x6)+(0.13x26)+(0.16x22)+(0.09x7)
=4.95+0.1+1.44+3.38+3.52+0.63
=14.02

The total cost obtained in HM is minimum when compared to other existing methods like NWC, HM, LCM.

Table 11: Comparison table

NWC Method	LCM Method	HEURISTIC Method	Harmonic mean method
15.21	14.29	14.71	14.02

7. Conclusion:

In this work, to solve PFTP, arithmetic and harmonic mean is introduced .The methods which we introduced here gives the better optimum solutions when we compare to other methods .In future, these methods can be applied to various applications of Pythagorean fuzzy problems which deals with real world problems.

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