

## Further Comments on the Explanation of the Latest Puzzling Observation of the 21 cm Radio Line from the Early Universe: the Role of the Singular Kind of Hydrogen Atoms

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ABSTRACT: In our previous papers (Oks 2018a, 2018b) - referred below as papers I and II, respectively - we brought to the attention of the astrophysical community the existence of a "singular kind of hydrogen atoms" (hereafter, SKHA) and its possible role in explaining a puzzling observational result published in Nature by Bowman et al (2018a). The existence of the SKHA was proven in Oks paper (2001) both theoretically and by the analysis of atomic experiments. Bowman et al (2018a) observed the absorption profile of the redshifted 21 cm line from the early Universe. They found that the amplitude of the profile was more than a factor of two greater than the largest predictions. This could mean that the primordial hydrogen gas was much cooler than expected. In the present paper we provide further details - compared to papers I and II - on the alternative explanation (based on the SKHA) of the puzzling observational results from Bowman et al paper (2018a). Namely, we note that the SKHA have not only the ground state, but also excited states, but all these states are S-states, i.e., the states of the zero orbital momentum. Therefore, the AKHA do not have excited discrete states that can be coupled to the ground state via the electric-dipole radiation. We also note that the continuum states of the SKHA are only the states of the zero orbital momentum. Therefore, the continuum states of the SKHA also cannot be coupled via the electric-dipole radiation. Then we show that the possible presence of the SKHA in the primordial gas would lower the excitation temperature of the hyperfine doublet (the spin temperature) by lowering the kinetic gas temperature to some effective value. We show quantitatively that this seems to be sufficient for explaining the puzzling observational results by Bowman et al (2018).

*Kew words:* singular kind of hydrogen atoms; explanation of the puzzle of 21 cm radio line; early Universe; cosmic microwave background

### 1. INTRODUCTION

In our previous papers (Oks 2018a, 2018b) – referred below as papers I and II, respectively – we brought to the attention of the astrophysical community the existence of a "singular kind of hydrogen atoms" (hereafter, SKHA) and its possible role in explaining a puzzling observational result published in Nature by Bowman et al (2018a). The existence of the SKHA was proven in Oks paper (2001) both theoretically and by the analysis of atomic experiments. Important details about the SKHA were briefly reiterated in paper I. The above word "singular" refers to the fact that the SKHA are described by the singular solution of the Dirac equation outside of the proton.<sup>1/</sup>

For the convenience of the readers, we reiterate here the introductory part of paper II. This study was motivated by the puzzling result by Bowman et al (2018a): they observed the 21 cm line (redshifted from the rest frequency of 1,240 MHz to the frequency of 78 MHz) from the early Universe. Bowman et al (2018a) observed the absorption profile of this line: namely, as hydrogen atoms absorb photons from the cosmic microwave background (CMB). (The underlying physical mechanism was the ultraviolet light from stars formed in the early Universe – the light that is expected to penetrate the primordial hydrogen gas and to alter the excitation of the hydrogen 21 cm hyperfine structure line.) Bowman et al (2018a) found that the amplitude of the profile was more than a factor of two greater than the largest predictions. This could mean that the primordial *hydrogen gas was much cooler than expected*, as noted by Bowman et al (2018a).

Hills et al (2018) expressed concerns about some aspects of the data processing by Bowman et al (2018a), though it was admitted by Hills et al (2018) that their analysis does not prove that the feature identified by Bowman et al. (2018a) is absent. In response, Bowman et al (2018b) pointed out that they conducted tests that showed that the recorded absorption signal was indeed astronomical (rather than having to do with the data processing). Bowman et al (2018b) also wrote that they have data that exclude some of the alternative signal models proposed by Hills et al (2018).

Several astrophysical explanations of the result by Bowman et al (2018a) were proposed in the literature. The first proposition was presented by Barkana (2018). He suggested that the additional cooling of the hydrogen gas was due to collisions with some kind of a dark matter. According to Barkana (2018), these dark-matter particle must be lighter than 4.3 GeV (meaning that they could have, e.g., the baryonic mass). Within the range of "lighter than 4.3GeV" Barkana did not provide any specificity about the dark matter he resorted to.

Feng and Holder (2018) proposed that the results by Bowman et al (2018a) could be explained by a high-z radio background supplementing the cosmic microwave background (CMB) as the illuminating backdrop. Ewall-Wice et al (2018) suggested that the additional radio background could arise from accretion onto growing black holes.

For completeness we note that Barkana's suggestion (2018) was criticized by Mirosha and Furlanetto (2019). They wrote that a weakly charged dark matter particle (capable of cooling the baryons through Rutherford scattering) cannot account for the signal observed by Bowman et al (2018a) without causing tension elsewhere. For example, Muñoz & Loeb (2018) estimated that if there is a charged dark matter particle, it can only constitute  $\sim 10$  per cent or less of all of the dark matter. Muñoz & Loeb (2018) suggested that the results by Bowman et al (2018a) could be explained if less than one per cent of the dark matter has a mini-charge, a million times smaller than the electron charge, and a mass in the range of 1–100 times the electron mass.

However, for fairness it should be clarified that Barkana (2018) himself wrote that the subcase of a weakly charged dark matter should be probably ruled out. Instead, Barkana (2018) assumed some kind of a non-standard Coulomb-like interaction between dark-matter particles and baryons that does not depend on whether the baryons are free or bound within atoms.<sup>2/</sup>

In the present paper we clarify the following. The SKHA have not only the ground state, but also excited states. However, all these states are S-states, i.e., the states of the zero orbital momentum. Therefore, the AKHA do not have excited discrete states that can be coupled to the ground state via the electric-dipole radiation. We also note that the continuum states of the SKHA are only the states of the zero orbital momentum. Therefore, the continuum states of the SKHA also cannot be coupled via the electric-dipole radiation.

Then we show that the possible presence of the SKHA in the primordial gas would lower the excitation temperature of the hyperfine doublet (the spin temperature) by lowering the kinetic gas temperature to some effective value. We show quantitatively that this seems to be sufficient for explaining the puzzling observational results by Bowman et al (2018).

# 2. FURTHER DETAILS ON THE SINGULAR KIND OF HYDROGEN ATOMS (SKHA) AND THE EXPERIMENTAL EVIDENCE OF THEIR EXISTENCE

In this section first we briefly summarize the results of Oks paper (2001). Solutions of the Dirac equation for the electron in the Coulomb field are common eigenfunctions of four operators (as it is well-known – see, e.g., the textbook by Rose (1961)) – the Hamiltonian H, the projection  $J_z$  of the total angular momentum, the square  $J^2$  of the total angular momentum, and the following operator:

$$\mathbf{K} = \beta(2\mathbf{L}\mathbf{s} + 1). \tag{1}$$

Here  $\beta$  is the Dirac matrix of the rank four, whose nonzero elements are  $\beta_{11} = \beta_{22} = 1$ ,  $\beta_{33} = \beta_{44} = -1$ ; **L** and **s** are the operators of the orbital angular momentum and spin, respectively; **Ls** denotes the dot-product (also known as the scalar product) of the latter two operators. Eigenvalues of the operators K and J<sup>2</sup> are connected as follows:  $k = \pm (j + 1/2)$ .

Hydrogen atoms in the stationary states have the following well-known energies

$$E_{Nk} = mc^{2} \{ 1 + \alpha^{2} / [N + (k^{2} - \alpha^{2})^{1/2}]^{2} \}^{-1/2},$$
(2)

where N is the radial quantum number. For the ground state, the quantum numbers N and k have the following values

$$N = 0, k = -1,$$
 (3)

so that

$$E_{0-1} = mc^2 (1 - \alpha^2)^{1/2}.$$
 (4)

For hydrogen atoms the radial part  $R_{Nk}(r)$  of the coordinate wave functions has the following behavior at small r (see, e.g., the textbook by Rose (1961)):

$$R_{Nk}(r) \propto 1/r^{1+s}, s = \pm (k^2 - \alpha^2)^{1/2}.$$
 (5)

For the ground state, Eq. (5) reduces to:

$$R_{0-1}(r) \propto 1/r^{q}, q = 1 \pm (1 - \alpha^2)^{1/2}.$$
 (6)

In Oks paper (2001) it was shown, that with the allowance for the finite proton size, both the regular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 + (1 - \alpha^2)^{1/2}$  are legitimate for the ground state. The corresponding derivation in Oks paper (2001) used *only* the fact that in the ground state the eigenvalue of the operator K is k = -1. Therefore, *actually the corresponding derivation from Oks paper (2001) is valid not just for the ground state, but for any state of hydrogenic atoms/ions characterized by the quantum number* k = -1. Those are S-states (l = 0), specifically  ${}^{2}S_{1/2}$  states. So, both the regular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 + (1 - \alpha^2)^{1/2}$  are legitimate not only for the ground state  ${}^{1}S_{1/2}$ , but also for the states  ${}^{2}S_{1/2}$ ,  ${}^{3}S_{1/2}$ , and so on, i.e., for the states  ${}^{n}S_{1/2}$ , where n = N + |k| = N + 1 is the principal quantum number (n = 1, 2, 3, ...). Both the regular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the singular exterior solution corresponding to  $q = 1 - (1 - \alpha^2)^{1/2}$  and the s

These theoretical results led to the possible existence of a singular kind of hydrogen atoms (SKHA), corresponding to the singular solution outside the proton. It should be emphasized that any n-state of the SKHA and the corresponding n- state of the usual hydrogen atoms differ by the wave functions, but not by the energy – the energy is the same.

Moreover, Oks (2001) paper presented also the first *experimental proof* of the existence of the SKHA. Namely, for many decades there was a long-standing mystery of the huge discrepancy between the experimental and previous theoretical results concerning the high-energy tail of the linear momentum distribution in the ground state of hydrogen atoms. The previous theories predicted the tail to scale with the linear momentum p as  $\sim 1/p^6$ , while the experiments pointed out to the scaling of  $\sim 1/p^k$  with the value of k close to 4. It was shown in Oks (2001) that the allowance for the SKHA eliminates this huge discrepancy. Thus, there were already both the theoretical and the experimental proofs of the existence of the SKHA.

# 3. POSSIBLE ALTERNATIVE EXPLANATION OF THE PUZZLING OBSERVATION OF THE 21 CM RADIO LINE FROM THE EARLY UNIVERSE

The possible existence of the SKHA could provide an alternative explanation of the puzzling observational result from Bowman et al paper (2018a). Bowman et al (2018a) observed that the amplitude of the profile was more than a factor of two greater than the largest predictions, meaning that the primordial hydrogen gas was possibly much cooler than expected.

The intensity of the observable 21 cm line from the early Universe is given as the brightness temperature  $T_B$ , which is a linear combination of the CMB temperature  $T_{CMB}$  and the spin temperature  $T_s$  (the latter being the excitation temperature of the hyperfine transition).

The standard expression for the spin temperature, as presented, e.g., in Field paper of year 1958 (see also, e.g., paper by Zaldarriaga et al (2014) and review by Furlanetto et al (2006)) is the following:

$$\Gamma_{\rm S} = (T_{\rm CMB} + y_{\rm c} T_{\rm K} + y_{\rm Ly} T_{\rm Ly}) / (1 + y_{\rm c} + y_{\rm Ly}).$$
(7)

Here the  $2^{nd}$  term in the numerator relates to the collisional excitation of the hyperfine transition, which couples  $T_s$  to the gas kinetic temperature  $T_K$ ,  $y_c$  being the corresponding coupling coefficient. The  $3^{rd}$  term in the numerator relates to the Wouthuysen-Field effect:  $T_{Ly}$  is the color temperature of the radiation field in the Lyman series and  $y_{Ly}$  is the corresponding coupling coefficient. Physically, the Wouthuysen-Field effect is the transition between the hyperfine structure sublevels of the ground state facilitated by the absorption and the subsequent reemission of a photon of the Lyman series – mostly the Ly-alpha photon.

The coupling coefficients in Eq. (7) are as follows:

$$y_{c} = C_{10}T_{*}/(A_{10}T_{K}), y_{1v} = P_{10}T_{*}/(A_{10}T_{1v}).$$
 (8)

Here  $C_{10}(T_K)$  is the collisional de-excitation rate of the triplet hyperfine sublevel (labeled 1) to the singlet hyperfine sublevel (labeled 0),  $T_* = 0.068K$ ,  $A_{10}$  is the corresponding Einstein coefficient,  $P_{10}$  is the direct de-excitation rate of the sublevel 1 due to absorption of an Ly $\alpha$  photon followed by the decay to sublevel 0.

Bowman et al (2018a) noted that the most intensive observed absorption signal corresponded to the redshift  $z \approx 17$ . Since the CMB temperature is  $T_{CMB} = 2.725(1 + z)$  K, than at  $z \approx 17$  there was  $T_{CMB} \approx 49$  K. According to the standard cosmology, at  $z \approx 17$  there was  $T_{K} \approx 7$  K, as noted by Barkana (2018). However, for explaining the anomalous brightness of the absorption signal observed in 2018 by Bowman et al (while the spin temperature  $T_{s}$  is given by Eq. (7)) the gas kinetic temperature  $T_{K}$  should not exceed 5.1 K, as also noted by Barkana (2018).

Our alternative explanation of the puzzling observational result from Bowman et al paper (2018) is the following. Let us follow the logic of Barkana (2018) paper, but with the substitution of an unspecified dark matter by the SKHA.

In distinction to usual hydrogen atoms, the SKHA do not have excited discrete states that can be coupled to the ground state via the electric-dipole radiation. (The SKHA still have two hyperfine sublevels of the ground state corresponding to the same 21 cm wavelength as usual hydrogen atoms.) This affects the spin temperature  $T_s$  as follows.

The SKHA decouple from the CMB *earlier* than usual hydrogen atoms. Indeed, the SKHA decouple from the CMB when, in the course of the Universe expansion, the CMB temperature drops to the value  $T_{CMB,S} = \alpha U_i$ , where  $U_i$  is the ionization potential of all kinds of hydrogen atoms and  $\alpha$  is a coefficient of the order  $10^{-1.5}$  (whose exact value is immaterial for the present reasoning because it will cancel out); the additional superscript S of  $T_{CMB,S}$  stands for SKHA. In distinction, the usual hydrogen atoms decouple from the CMB at  $T_{CMB,U} = \alpha E_{21}$ , where  $E_{21} = 3U_i/4$  is the energy difference between the first excited and ground states; the additional superscript U of  $T_{CMB,U}$  stands for usual hydrogen atoms. To visualize: as the CMB temperature drops from  $T_{CMB,S}$  to  $T_{CMB,U}$ , the CMB can still radiatively couple numerous discrete excited states of usual hydrogen atoms to the ground state and then at  $T_{CMB} < T_{CMB,U}$  there are no more excited states to be radiatively coupled to the ground state. For the SKHA already at  $T_{CMB} < T_{CMB,S}$  there are no discrete excited states that can be coupled to the ground state via the electric-dipole radiation. Obviously,  $T_{CMB,U}/T_{CMB,S} = E_{21}/U_i = 3/4$ .

Let us denote by  $a_1$  the value of the expansion parameter a of the Universe at the SKHA decoupling from the CMB, i.e., at  $T_{CMB,S}(a_1) = \alpha U_i$ . Obviously, the kinetic gas temperature  $T_{K,S}(a_1)$  of the SKHA at  $a = a_1$  is equal to  $T_{CMB,S}(a_1)$ , so that  $T_{K,S}(a_1) = \alpha U_i$ .

Let us denote by  $a_2$  the value of the expansion parameter of the Universe at the decoupling of usual hydrogen atoms from the CMB, i.e., at  $T_{CMB,U}(a_2) = \dot{a}E_{21}$ . Obviously, the kinetic gas temperature  $T_{K,U}(a_2)$  of usual hydrogen atoms at  $a = a_2$  is equal to  $T_{CMB,S}(a_2)$ , so that  $T_{K,S}(a_2) = \dot{a}E_{21}$ .

As the SKHA decouple from the CMB, their kinetic gas temperature  $T_{K,S}$  evolves proportional to  $1/a^2$  (assuming an adiabatic expansion for simplicity), so that  $T_{K,S} = C/a^2$ , where C is some coefficient. Therefore,  $T_{K,S}(a_2)/T_{K,S}(a_1) = (a_1/a_2)^2$ . As for the CMB temperature, it evolves proportional to 1/a, so that  $T_{CMB}(a_2)/T_{CMB}(a_1) = a_1/a_2$ . Consequently, by using relations  $T_{K,S}(a_1) = T_{CMB}(a_1)$  and  $T_{K,U}(a_2) = T_{CMB}(a_2)$ , for the ratio  $T_{K,S}(a_2)/T_{K,U}(a_2)$  one obtains:

$$T_{K,S}(a_2)/T_{K,U}(a_2) = T_{K,S}(a_2)/T_{CMB}(a_2) = [T_{K,S}(a_2)/T_{K,S}(a_1)][T_{CMB}(a_1)/T_{CMB}(a_2)] = (a_1/a_2)^2(a_2/a_1) = a_1/a_2.$$
(9)

Since  $a_1/a_2 = T_{CMB}(a_2)/T_{CMB}(a_1) = E_{21}/U_i$ , the final result for the above ratio is:

$$\Gamma_{K,S}(a_2)/T_{K,U}(a_2) = E_{21}/U_i = 3/4.$$
(10)

Thus, at  $a = a_2$ , the SKHA fluid is colder than the fluid of usual hydrogen atoms. At some  $a > a_2$ , the two fluids come to the thermal equilibrium with each other (due to the scattering of the usual hydrogen atoms with the SKHA), so that their effective (final) kinetic temperature is as follows<sup>3/</sup>

$$T_{K,eff} = (T_{K,U}n_U + T_{K,S}n_S)/(n_U + n_S) = (T_{K,U} + T_{K,S}n_S/n_U)/(1 + n_S/n_U) = (11)$$
  
=  $T_{K,U}[1 + (3/4) n_S/n_U]/(1 + n_S/n_U) = T_{K,U}[1 + (3/4) (\rho_S/\rho_U)\mu_U/m_S]/[1 + (\rho_S/\rho_U)\mu_U/m_S],$ 

where  $n_U$  and  $n_s$  are the corresponding number densities,  $\rho_U$  and  $\rho_s$  are the corresponding mass densities,  $\mu_U$  is the mean molecular mass of the (usual) neutral primordial gas,  $m_s$  is the atomic hydrogen mass ( $m_s = 0.939$  GeV). By using the same numerical values as employed by Barkana (2018) (see, e.g., Eq. (3) from his paper), Eq. (11) can be represented in the form:

$$T_{\text{Keff}} \approx T_{\text{KU}} [1 + (3/4)(6 \text{ GeV})/\text{m}_{\text{S}}] / [1 + (6 \text{ GeV})/\text{m}_{\text{S}}] \approx 0.79 \text{ T}_{\text{KU}}.$$
 (12)

Consequently, with the allowance for the possible SKHA, at the redshift  $z \approx 17$ , the effective kinetic gas temperature would be lower than the lowest possible kinetic gas temperature  $T_{K,U} \approx 7$  K in the standard scenario. Namely, it would be  $T_{K,eff} \approx 0.79$   $T_{K,U} \approx 5.5$  K. This temperature is much closer to the threshold estimated as  $\approx 5.1$  K (required for explaining the observations by Bowman et al (2018) in the standard scenario), than  $T_{K,U} \approx 7$  K. In detail, while  $T_{K,U} \approx 7$  K exceeded 5.1 K by more than 37%, the effective temperature  $T_{K,eff} \approx 5.5$  K exceeds 5.1 K only by less than 8%, which it within the error margin of the estimated value of  $T_{K,eff}$ .

Thus, the lowering of the kinetic gas temperature to the effective value of  $0.79T_{K,U}$  seems to be sufficient for explaining the observations by Bowman et al (2018a).

### 4. CONCLUSIONS

It should be emphasized upfront that this paper is not intended as a search for an additional (astrophysical) evidence of the existence of the SKHA – there is already the experimental evidence of their existence based on the analysis of atomic experiments (as presented briefly in the above Sect. 2 and presented in detail in Oks (2001) paper). Instead, in this paper we explored a "what if" scenario: what if in place of some unspecified dark matter resorted to by Barkana (2018) for explaining the observations by Bowman et al (2018a), one would consider the SKHA.

We showed that in this scenario the possible presence of the SKHA would lower the kinetic gas temperature to some effective value. This seems to be sufficient for explaining the puzzling observational results by Bowman et al (2018).

This explanation seems to be more specific and natural than adopting a possible cooling of barions either by unspecified dark matter particles, as in paper by Barkana (2018), or by some exotic dark matter particles of the charge of the million times smaller than the electron charge, as in paper by Muñoz & Loeb (2018). Also our explanation does not require an additional radio background suggested by Feng & Holder (2018) and by Ewall-Wice et al (2018).

It should be noted (as in paper II) that this alternative explanation of the Bowman et al (2018) results does not require or depend on the terminology introduced in paper I, such as "two flavors of hydrogen atoms" and "isohydrogen spin (isohyspin)". The explanation of the Bowman et al (2018) results presented here stands on its own.

Further observational studies of the redshifted 21 cm radio line from the early Universe could help to find out which explanation is the most relevant.

#### Notes

- 1. Here and below, by "singular" we mean the strongly-singular solution of the Dirac equation for the Coulomb field in distinction to the commonly accepted "regular" solution that has a weak singularity at the origin.
- 2. Our paper should not be construed as a criticism of Barkana (2018) paper: we greatly appreciate his paper and use some numerical estimates from it.
- 3. Because the SKHA have only S-states and the S-states are meta-stable, a significant share of the SKHA are in the excited  $n^2S_{1/2}$  states, possibly including  $n \gg 1$  (in distinction to the usual hydrogen atoms). Since the characteristic size of these states scales as  $\sim n^2$ , so that the collisional cross-section scales as  $\sim n^4$ , then for the SKHA collisions are much stronger than for the usual hydrogen atoms.

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