DOI : https://doi.org/10.56452/6-1-27

ISSN: 0974-5823 Vol. 6 No. 1 June, 2021

International Journal of Mechanical Engineering

The Theoretical Study of Evolution of Linearized Perturbations in a Magnetohydrodynamic Bounded Couette Flow

Vijayalakshmi A.R.

Department of Mathematics, Maharani's Science College for Women,
Bengaluru–560 001. INDIA. E-mail: drarv@rediffmail.com

Abstract:
Initial value problems are used to look into the development of linearized perturbations in a magnetohydrodynamic bounded couette flow. The Fourier transform is applied to a point source of transverse velocity and magnetic field to get an equation in time that is then solved for the Fourier amplitudes. For small amounts of Alfve′n velocity, perturbation solutions may be found. The velocity and magnetic field plots are drawn for different values of Alfve′n velocity.

Key words: Magnetohydrodynamic bounded couette flow, Alfve′n velocity, Fourier transform, point source of transverse velocity, point source of magnetic field.

Introduction

Astrophysicists and geophysicists are both interested in the stability of shear flow that conducts electricity. The study of the stability of such flows is crucial to our capacity to deduce the nature of physical processes in the universe as a whole.

Afroja Parvin et al.[1] looked into the effects of Hall current on a conducting, viscous, incompressible fluid wedged between two parallel, non-conducting surfaces.Both the fluid velocity and the magnetic field are constant and perpendicular to the top plate. The top plate is moving at a steady speed while the bottom plate is fixed. Using the implicit finite difference method, they solved the governing non-linear coupled partial differential equations that were produced. In the presence of a transverse magnetic field, Muhim Chutia et al.[6] investigated numerically the unsteady hydromagnetic couette flow of a viscous, incompressible, and electrically conducting fluid in a rotating system between two infinitely long parallel porous plates. The study of stability of a simple shear flow of an incompressible fluid with a piecewise linear velocity profilethe influence of magnetic field was published by Ruderman and Brevdo [7]. A transitional magnetic-free layer is sandwiched between two semi-infinite areas in the flow. They came to the conclusion that the strength of the interstellar magnetic field needed to stabilise this section of the heliopause is one order of magnitude higher when a transitional layer is included in the model compared to when the heliopause is modelled as a tangential discontinuity.

By assuming a uniform mean magnetic field, Stuart [8] studied the stability of Poiseuille flow in a flat plane. The effect of a coplanar magnetic field on the equilibrium of a conducting fluid moving between parallel planes was investigated by Hains[3]. Numerical results were obtained for the scenario when the initial perturbations to the magnetic field disappear, following the derivation of the general stability equation for small magnetic Reynolds number. When a homogeneous magnetic field is oriented perpendicular to the flow and of appropriate strength, Hunt [4] shown that the most unstable disturbance has a wave number vector that is not perpendicular to the flow, indicating that it exists in three dimensions. For unbounded plane couette flow, Lerner and Knobloch [5] used the separation of variables approach to investigate the stability of dissipative magnetohydrodynamic shear flow in a parallel magnetic field. Both the molecular viscosity and the finite conductivity were shown to be stabilising factors. In their work on the development of general three-dimensional perturbations in a magnetohydrodynamic couette flow, Vijayalakshmi and Balagondar [9] found graphically that for varying values of Alfven velocity, the behaviour of the total energy and the sum of the first five components of energy are qualitatively similar. Researchers Zakir Hussain et al. [10] found that a transverse magnetic field of a certain magnitude destabilises Couette flow, while a field of a different magnitude stabilises the flow when applied to an electrically conducting fluid in a magnetohydrodynamic flow between parallel plates using the Chebyshev collocation method.

This research is an extension of work by Criminale and Drazin [2] to magnetohydrodynamic couette flow with velocity and magnetic field pulse. As a function of all space variables and time, the full general solution to the linearized equations of motion is found. Rotational and irrotational aspects of the disturbances are separated. Because the mean flow is unbounded yet corresponds with the actual flow in the layer, the rotational solution is the answer to the hypothetical initial-value issue. When the boundary conditions and the interfacial conditions at a wall are met, a unique irrotational solution is defined for each layer.

Mathematical Formulation

We consider an electrically conducting fluid of density ρ, moving with velocity $\vec{q}$ in the presence of a magnetic field $\vec{H}$. Gravity is ignored. For a fluid that really is incompressible, inviscid, magnetohydrodynamic, and Boussinesq, the governing equations are

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\nabla.\vec{H} = 0, \tag{2}$$

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right) = -\nabla P + \mu_m (\vec{H}.\nabla)\vec{H}, \tag{3}$$

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q}.\nabla)\vec{H} = (\vec{H}.\nabla)\vec{q}, \tag{4}$$

where $P = p + \frac{\mu_m H^2}{2}$ is the total pressure, μ_m is the magnetic permeability.

The ambient state of the system is

$$\vec{q}_0 = (U(y) = \sigma y, 0, 0),\ \vec{H}_0 = (H_0, 0, 0),\ P = P_0(y) \tag{5}$$

where σ represents the mean shear of the flow. It is assumed that σ and H_0 are constants. In the theory of linear stability, we disrupt the mean flow by a small bit, like a wave, and then superimpose this new flow over mean flow.

$$\vec{q} = \vec{q}_0 + \vec{q}',\ \vec{H} = \vec{H}'_0 + \vec{H}',\ P = P_0 + P', \tag{6}$$

DOI : https://doi.org/10.56452/6-1-27

where $\vec{q}'$, $\vec{H}'$, P' are the variables that experienced a change in velocity, magnetic field, and pressure.

By (i) using a transformation based on shifting coordinates,

$$T = t,\ \xi = x - \sigma y t,\ \eta = y,\ \zeta = z \tag{7}$$

(ii) using three - dimensional Fourier transformation given by

$$\hat{u}(\alpha;\beta;\gamma;T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(\xi;\eta;\zeta;T)\, e^{i(\alpha\xi+\beta\eta+\gamma\zeta)}\, d\xi\, d\eta\, d\zeta \tag{8}$$

with similar expressions for $\hat{v}, \hat{w}$, $\hat{H}_x, \hat{H}_y, \hat{H}_z$ and $\hat{P}$ (iii) making use of Squire transformation defined by $\bar{\alpha} = \left(\alpha^2+\gamma^2\right)^{1/2}$ and $\varphi = \arctan\left(\gamma/\alpha\right)$, the velocity and magnetic field components in the $\bar{\alpha}$ and φ directions are given by

$$\bar{u} = \frac{\alpha\hat{u}+\gamma\hat{w}}{\bar{\alpha}},\ \bar{w} = \frac{-\gamma\hat{u}+\alpha\hat{w}}{\bar{\alpha}},\ \bar{H}_x = \frac{\alpha\hat{H}_x+\gamma\hat{H}_z}{\bar{\alpha}},\ \bar{H}_z = \frac{-\gamma\hat{H}_x+\alpha\hat{H}_z}{\bar{\alpha}}. \tag{9}$$

The linearized equations of motion with Boussinesq approximation and omitting the primes becomes

$$\frac{d}{dT}\left(K^2\hat{v}\right) + i\alpha V_A^2 K^2 \hat{H}_y = 0 \tag{10}$$

$$\frac{d\hat{H}_y}{dT} = -i\alpha\hat{v}. \tag{11}$$

where $V_A^2 = \frac{\mu_m H_0^2}{\rho_0}$, V_A is the Alfve′n velocity, ρ_0 is the equilibrium density.

$K^2 = \bar{\alpha}^2 + (\beta - \sigma\alpha T)^2$ and $\bar{\alpha}^2 = \left(\alpha^2+\gamma^2\right)$.

By taking the divergence of the momentum equations, it is found that the pressure amplitude $\hat{P} = \frac{-i2\sigma\alpha\hat{v}}{K^2}$ when $K^2 \neq 0$. Equation (10) admits two sets of solutions for $\hat{v}$, first, for $K^2 \neq 0$, when the disturbance is rotational and second for $K^2 = 0$, when the disturbance is irrotational as the vanishing of the product $K^2\hat{v}$ is equivalent to Laplace equation $\nabla^2\hat{v} = 0$ in real space. But for $\hat{H}_y$ solution exists for only $K^2 \neq 0$, since for $K^2 = 0$, $K^2\hat{H}_y = 0$ corresponds to $\nabla^2\hat{H}_y = 0$ which is equivalent to $\nabla x \vec{H} = 0$ and $\nabla.\vec{H} = 0$, which implies that $\vec{H}$ is force free magnetic field i,e., there is no magnetic field. Hence $\nabla^2\hat{H}_y = 0$ which corresponds to irrotational solution, is not taken into account.

Now considering the case $K^2 \neq 0$, we assume perturbation solution for $\hat{v}_R$, the rotational component of $\hat{v}$ for small values of V_A^2 (Alfve′n velocity) in the form

$$\hat{v}_R(\alpha,\beta,\gamma,T) = \hat{v}_0(\alpha,\beta,\gamma,T) + V_A^2\,\hat{v}_1(\alpha,\beta,\gamma,T) + \left(V_A^2\right)^2 \hat{v}_2(\alpha,\beta,\gamma,T) + \ldots \tag{12}$$

$$\hat{H}_y(\alpha,\beta,\gamma,T) = \hat{H}_{y0}(\alpha,\beta,\gamma,T) + V_A^2\,\hat{H}_{y1}(\alpha,\beta,\gamma,T) + \left(V_A^2\right)^2 \hat{H}_{y2}(\alpha,\beta,\gamma,T) + \ldots \tag{13}$$

DOI : https://doi.org/10.56452/6-1-27

We find that

$$\hat{v}_0 = \frac{\hat{\Omega}_0(\alpha,\beta,\gamma)}{\bar{\alpha}^2 + (\beta - \sigma\alpha T)^2}, \tag{14}$$

$$\hat{v}_1 = \left\{ -\frac{\hat{\Omega}_0\bar{\alpha}^3}{\sigma}\left[\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right) - \frac{1}{3}\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right) + \frac{1}{3}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^3\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right) \right.\right.$$

$$\left.\left. -\frac{1}{6}\left(\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\right)^2\right] + \frac{i\bar{\alpha}^3\hat{\Omega}_1}{\sigma}\left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right) + \frac{1}{3}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^3\right)\right\}\frac{1}{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}, \tag{15}$$

$$\hat{v}_2 = \left\{\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^3\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\left(\frac{7\bar{\alpha}^5\hat{\Omega}_0}{18\sigma^3\alpha}\right) + \left(-\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right) + \left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^2\right)\right.$$

$$\left(\frac{5\bar{\alpha}^5\hat{\Omega}_0}{36\sigma^3\alpha}\right) + \left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^5\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)\left(\frac{\bar{\alpha}^5\hat{\Omega}_0}{10\sigma^3\alpha}\right) + \left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^4\left(\frac{\bar{\alpha}^5\hat{\Omega}_0}{6\sigma^3\alpha}\right)$$

$$+\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^3\left(-\frac{2\bar{\alpha}^5\hat{\Omega}_0}{3\sigma^3\alpha} + \frac{5i\alpha\bar{\alpha}^5\hat{\Omega}_1}{27\sigma^2}\right) + \left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\left(-\frac{2\bar{\alpha}^5\hat{\Omega}_0}{\sigma^3\alpha} - \frac{8i\alpha\bar{\alpha}^5\hat{\Omega}_1}{9\sigma^2}\right)$$

$$+\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\left(-\frac{2\bar{\alpha}^5\hat{\Omega}_0}{\sigma^3\alpha} + \frac{8i\alpha\bar{\alpha}^5\hat{\Omega}_1}{9\sigma^2}\right) + \left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\tan^{-1}\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)\left(\frac{2\bar{\alpha}^5\hat{\Omega}_0}{3\sigma^3\alpha}\right)$$

$$+\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^5\left(\frac{i\alpha\bar{\alpha}^5\hat{\Omega}_1}{\sigma^2}\right) + \left(\left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right) + \left(\frac{\beta-\sigma\alpha T}{\bar{\alpha}}\right)^3\right)\log\left(\frac{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}{\bar{\alpha}^2}\right)$$

$$\left.\left(\frac{2i\,\alpha\bar{\alpha}^5\hat{\Omega}_1}{3\sigma^2}\right)\right\}\frac{1}{\bar{\alpha}^2+(\beta-\sigma\alpha T)^2}, \tag{16}$$

In the case of $K^2 = 0$, we apply the two-dimensional Fourier transform to the perturbation equations.

Using moving co-ordinate transformation, $K^2\hat{v} = 0$ corresponds to

$$\frac{\partial^2 \breve{v}_I}{\partial\eta^2} + 2i\sigma\alpha T\frac{\partial\breve{v}_I}{\partial\eta} - \left(\bar{\alpha}^2 + \sigma^2\alpha^2T^2\right)\breve{v}_I = 0, \tag{17}$$

$$\breve{v}_I = \breve{v}_I(\alpha,\eta,\gamma;T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_I(\xi,\eta,\varsigma,T)e^{i(\alpha\xi+\gamma\zeta)}\,d\xi\,d\zeta, \tag{18}$$

is the irrotational part of v. Equation (17), when solved, gives

$$\breve{v}_I = A(T)e^{-\bar{\alpha}\eta - i\sigma\alpha T\eta} + B(T)e^{\bar{\alpha}\eta - i\sigma\alpha T\eta}. \tag{19}$$

where the constants of integration, A(T) and B(T), are being used.

In order to combine $\hat{v}_R$ and $\breve{v}_I$ to obtain the complete solution and satisfy the matching condition $\hat{v}_R$ must be inverted once to obtain $\breve{v}_R(\alpha,\eta,\gamma;T)$i,e.,

$$\breve{v}_R(\alpha,\eta,\gamma;T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{v}_R(\alpha,\beta,\gamma;T)\,e^{-i\beta\eta}\,d\beta. \tag{20}$$

The initial velocity and initial magnetic field are given by

$$v(x,y,z,0) = V_0\,\delta(x-x_0)\delta(y-y_0)\delta(z-z_0). \tag{21}$$

$$H_y(x,y,z,0) = H_0\,\delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \tag{22}$$

Coordinate transformation and Fourier transform in three dimensions gives

$$\breve{v}_0(\alpha,\beta,\gamma) = \Omega_0(\alpha,\beta,\gamma) = V_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)}. \tag{23}$$

$$\breve{H}_{y_0}(\alpha,\beta,\gamma) = \Omega_1(\alpha,\beta,\gamma) = \tilde{H}_0 e^{i(\alpha x_0+\beta y_0+\gamma z_0)} \tag{24}$$

$\breve{v}_R$ is found to be

$$\breve{v}_R = e^{i(\alpha x_0+\gamma z_0-\sigma\alpha T\bar{\eta})}\Bigg\{V_0\left(1-\frac{V_A^2\bar{\alpha}^3}{6\sigma\alpha}-\frac{V_A^4\bar{\alpha}^5}{36\sigma^3\alpha}\right)e^{-\bar{\alpha}|\bar{\eta}|}+V_0\left(\frac{2V_A^2\bar{\alpha}^3}{3\sigma}+\frac{5V_A^4\bar{\alpha}^5}{18\sigma^3\alpha}\right)$$

$$\int_{-\infty}^{\infty}\frac{\eta' e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\bar{\eta}|}}{(\bar{\eta}-\eta')}\,d\eta' + V_0\left(\frac{V_A^2\bar{\alpha}^3}{3\sigma}-\frac{5V_A^4\bar{\alpha}^5}{36\sigma^3\alpha}\right)\int_{-\infty}^{\infty}\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\bar{\eta}|}}{\eta'}\,d\eta'$$

$$+\left(\frac{2i\bar{\alpha}^3\tilde{H}_0}{3\sigma}-\frac{4\bar{\alpha}^5V_0}{3\sigma^3\alpha}+\frac{46i\,\alpha\bar{\alpha}^3\tilde{H}_0}{27\sigma^2}\right)\left(\frac{-i\bar{\eta}}{2}e^{-\bar{\alpha}|\bar{\eta}|}\right)+\left(-\frac{2\bar{\alpha}^5V_0}{\sigma^3\alpha}+\frac{8i\alpha\bar{\alpha}^5\tilde{H}_0}{9\sigma^2}\right)$$

$$\left(\int_{-\infty}^{\infty}-i\frac{e^{-\bar{\alpha}(\bar{\eta}-\eta')-\bar{\alpha}\eta'}}{\eta'}d\eta'\right)-\left(\frac{\bar{\alpha}^5V_0}{10\sigma^3\alpha}-\frac{4i\alpha\bar{\alpha}^5\tilde{H}_0}{9\sigma^2}\right)\left(\int_{-\infty}^{\infty}i\frac{e^{-\bar{\alpha}|\bar{\eta}-\eta'|-\bar{\alpha}|\eta'|}}{2(\bar{\eta}-\eta')}d\eta'\right)\Bigg\}. \tag{25}$$

Now the complete solution will be

$$\breve{v} = \breve{v}_R + \breve{v}_I. \tag{26}$$

$\breve{v}_R$ and $\breve{v}_I$ are given by equations (25) and (19).

Magnetohydrodynamic bounded plane couette flow

Here, we focus on the example of a magnetohydrodynamic couette flow in a flat plane as in Fig.1. Here velocity $\breve{v}$ vanishes at $\eta = \pm H$, hence we have

$$e^{-\bar{\alpha}H-i\sigma\alpha TH}A + e^{\bar{\alpha}H-i\sigma\alpha TH}B = -\left[\breve{v}_R\right]_{\eta=+H} \tag{27}$$

$$e^{-\bar{\alpha}H-i\sigma\alpha TH}A + e^{\bar{\alpha}H-i\sigma\alpha TH}B = -\left[\breve{v}_R\right]_{\eta=+H} \tag{28}$$

From equations (27) and (28), A and B are found to be

DOI : https://doi.org/10.56452/6-1-27

$$A=\frac{\breve{v}_R(+H)e^{-\bar{\alpha}H+i\sigma\alpha TH}-\breve{v}_R(+H)e^{\bar{\alpha}H-i\sigma\alpha TH}}{2\sinh(2\bar{\alpha}H)}. \qquad (29)$$

$$B=\frac{\breve{v}_R(-H)e^{-\bar{\alpha}H-i\sigma\alpha TH}-\breve{v}_R(-H)e^{\bar{\alpha}H+i\sigma\alpha TH}}{2\sinh(2\bar{\alpha}H)}. \qquad (30)$$

where $\breve{v}_R(\pm H)=-\lfloor\breve{v}_R\rfloor_{\eta=\pm H}$.

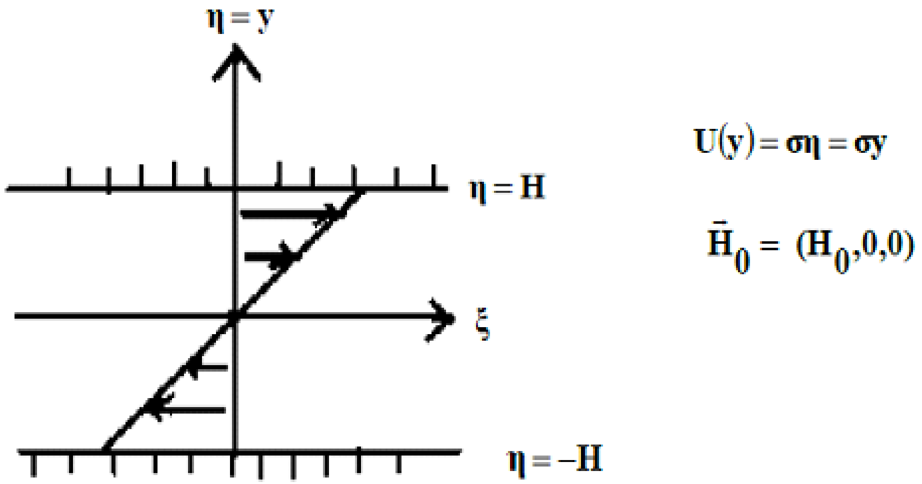

Fig. 1 **Sketch of bounded magnetohydrodynamic bounded Couette flow**

It is found that

$$\breve{v}_R(+H)=A_1\,e^{i\left(\alpha x_0+\gamma z_0-\sigma\alpha T\left(H-y_0\right)\right)}. \qquad (31)$$

$$\breve{v}_R(-H)=A_2\,e^{i\left(\alpha x_0+\gamma z_0-\sigma\alpha T\left(H-y_0\right)\right)}. \qquad (32)$$

$A_1=\breve{v}_R\big|_{\bar{\eta}=H-y_0},\ A_2=\breve{v}_R\big|_{\bar{\eta}=-(H-y_0)}$

DOI : https://doi.org/10.56452/6-1-27

Results and Discussion

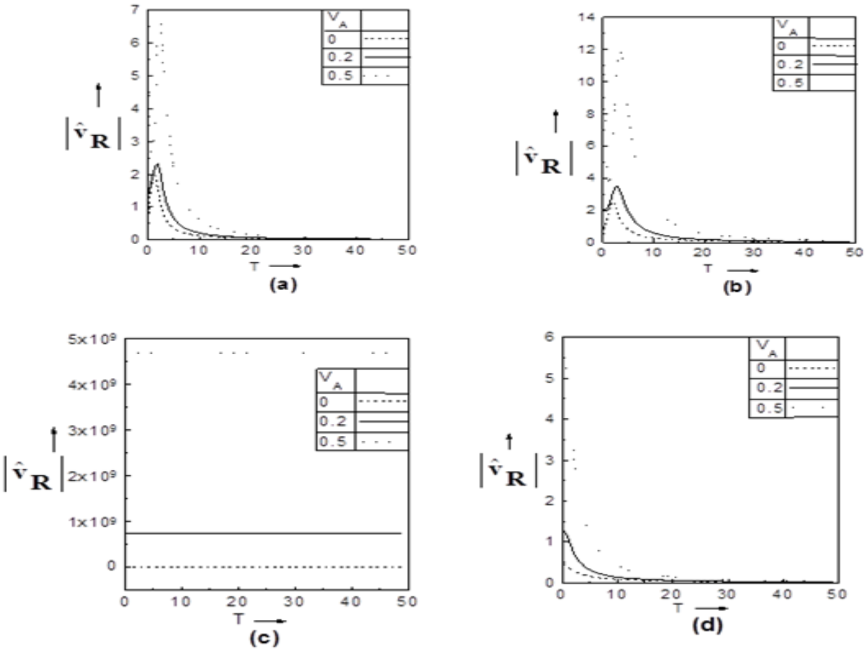

Fig 2. Curves of $\left|\hat{v}_{\mathbf{R}}\right|$ versus T for (a) $\varphi = 0^{\circ}$, (b) $\varphi = 45^{\circ}$, (c) $\varphi = 90^{\circ}$ and (d) $\varphi = 180^{\circ}$ for different values of $\mathbf{V_A}$.

Using a unit pulse of velocity and magnetic field as starting distributions, we have investigated the development of linearized perturbations in a bounded couette flow subject to inviscid magnetohydrodynamics. We have separated the rotational and irrotational parts of the perturbations. Plots are drawn to observe the variation of amplitude of rotational velocity $\left|\hat{v}_R\right|$ and magnetic field $\left|\hat{H}_y\right|$ with time.

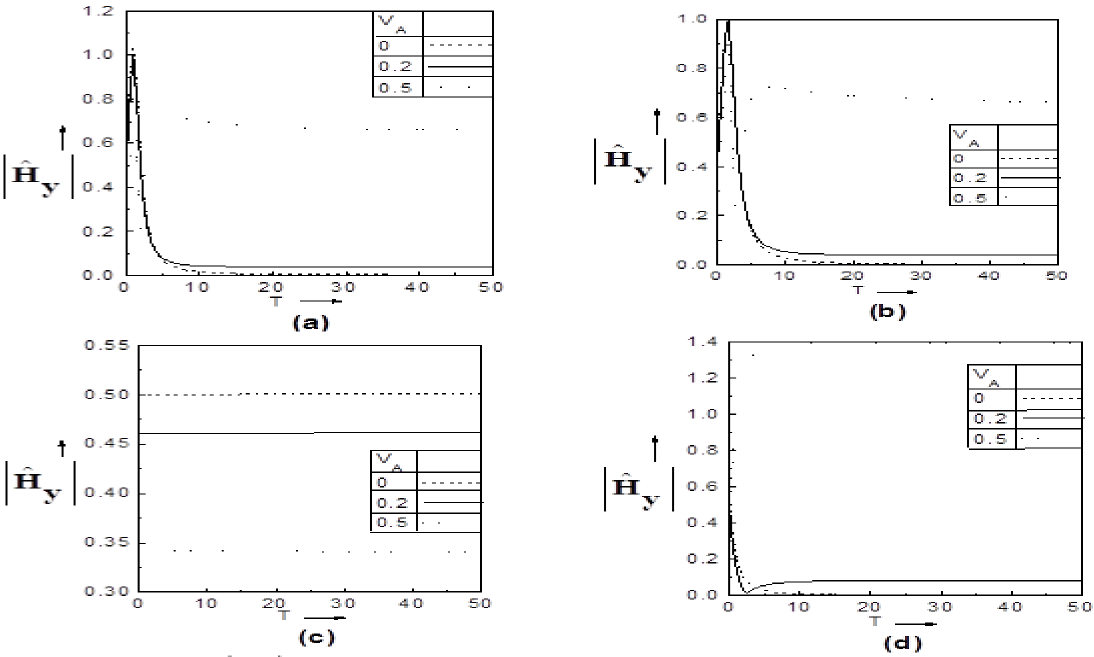

Fig. 3 Curves of $\left|\hat{H}_y\right|$ versus T for (a) $\varphi = 0^{0}$, (b) $\varphi = 45^{0}$, (c) $\varphi = 90^{0}$ and (d) $\varphi = 180^{0}$ for different values of V_A

Figs.2 (a)-(d) are plots of $\left|\hat{v}_R\right|$ versus T for different values of V_A (V_A = 0, 0.2, 0.5) and φ ($\varphi = 0^0, 45^0, 90^0, 180^0$). For $\varphi = 0^0, 45^0, 180^0$ as time increases there is decay in $\left|\hat{v}_R\right|$. Figs. 3 (a)-(d) are plots of $\left|\hat{H}_y\right|$ versus T for different values of V_A and for different values of φ ($\varphi = 0^0, 45^0, 90^0, 180^0$). At $\varphi = 0^0, 45^0, 180^0$, we see that with an increase in V_A, $\left|\hat{H}_y\right|$ decays with time. At $\varphi = 90^0$ there is no variation in velocity and magnetic field with time

Conclusions

Graphically it is found that as time elapses the perturbations die down and makes the flow stable. But for $\varphi = 90^0$ the perturbations neither dies down nor grows. This happens in the case of bounded flows where the boundaries are placed far away from each other (like oceans) which may lead to natural disasters. When there is no magnetic field present, these findings correlate well with those of Criminale and Drazin [2].

References

[1] Afroja Parvin, Tanni Alam Dola, and Md. Mahmud Alam , MHD Couette flow of viscous incompressible fluid with Hall current and suction, AIP Conf. Proc., 1754,040006, https://doi.org/10.1063/1.4958366, 2016.

[2] Criminale W O and Drazin P G, The evolution of linearized perturbations of parallel flows, Stud. App. Math., Vol. 83, pp.123-157, 1990.

[3] Hains F. D., Stability diagrams for magnetogasdynamic channel flow, Phys. Fluids, Vol. 8, no. 11, pp. 2014-2019, 1965.

[4] Hunt J.C.R., On the stability of parallel flows with parallel magnetic field, Proc. R. Soc. Lond. A, Vol. 293, pp. 342-358, 1966.

[5] Lerner J. and Knobloch E., The stability of dissipative magnetohydro-dynamic shear flow in a parallel magnetic field, Geophysics and Astrophysics, Fluid Dynm., Vol. 33, pp. 295-314, 1985.

[6] Muhim Chutia, Tabendra Nath Das and Pranob Jyoti Chetia, Numerical Solution of unsteady hydromagnetic couette flow in a rotating system bounded by porous plates with hall effects, Inter. J. Comp. Appls., Vol. 171, no.2, pp.1-6, 2017.

[7] Ruderman M.S., and Brevdo L. , Stability of an MHD shear flow with a piecewise linear velocity profile, Astro. and Astrophys., Vol. 448, no.3, pp.1177-1184, 2006.

[8] Stuart J. T., On the stability of viscous flow between parallel planes in the presence of co – planar magnetic field", Proc. R. Soc. Lond. A vol 221,pp 189-206,1954.

[9] Vijayalakshmi, A R and Balagondar, P M, The evolution of general three-dimensional perturbations in a magnetohydrodynamic couette flow, Inter. J. Appl. Mech. and Engg., Vol. 13, no.3, pp. 627-638, 2009.

[10] Zakir Hussain, Sultan Hussain, Tiantian Kong and Zhou Liu, "Instability of MHD couette flow of an electrically conducting fluid", AIP Advances, Vol 8, no.10, 105209, pp. 1-11,2018.