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SOFT NEAR RELATIONS IN SOFT TOPOLOGY

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Abstract

Interior and closure operators in topological spaces have applications to data reduction and GIS. Two sets in a topological space may have the same interior. For example the set Z of all integers and the set Q of all rational numbers have the same interior in the Euclidean topology of real numbers. Such sets are nearer to each other. The near relation on subsets of a topological space was studied by Thamizharasi in 2009. Recently Chitralekha et.al. studied the weak and strong forms of near relation in topological spaces. In this paper, the near relation has been discussed in soft topological settings.

Keywords:*soft topology*, *soft open*, *soft closed*, *soft interior*, *soft closure*

1.Introduction

In most of the real life problems, vage and rough concepts are ineluctable. Researchers used fuzzy sets and rough sets to solve the problems involving such vague concepts. Molodtsov[8] initiated the study of soft sets in 1999. Following this several mathematicians studied the applications of soft sets . Muhammad Shabir and Munazza Naz[9] introduced the notion of soft topology. Many topologists[1,2,3,5,6,7,10,11,12,13] concentrated their research in soft topology by extending some of the recent concepts in general topology to soft topology and applications of soft sets. Recently Chitralekha et.al.[4] studied several types of near relations in topological spaces. In this paper, the concept of soft near relation is introduced and has been discussed.

1.1 Preliminaries

Thamizharasi[14] introduced the notion of near relation in topology. If (X, τ) is a topological space, A and B are subsets of X then A is near to B if *Int*A = *Int*B. Throughout this paper, X is a non empty set and E is a parameter space. By a soft subset F of X with parameter space E, we mean a function $F:E\rightarrow 2^X$. Let S(X, E) be the collection of all soft subsets of X with parameter space E. Muhammad Shabir and Munazza Naz[9] introduced the notion of soft topology on S(X,E). For the basic concepts and results, the reader may consult[1,2,3,4,6,8,9,10,11,12,13].

Let F and G be soft subsets in S(X,E). We use the following notations.

$F\sqsubseteq G$ means F is a soft subset of G.

$F\sqsupseteq G$ means F is a soft superset of G.

$F\sqcap G$ means soft intersection between F and G.

$F\sqcup G$ means soft union between F and G.

$F\boxminus G$ means soft difference G from F.

$X_E \boxminus F$ means the soft complement of F.

1.2 Definition

Let τ be a sub collection of S(X, E). Then τ is said to be a soft topology[9] on X if

(i) $\varnothing_E$ and X_E belong to τ.

(ii) τ is closed under finite soft intersection.

(iii) τ is closed under arbitrary soft union.

If τ is a soft topology on X then the triplet (X, τ, E) is called a soft topological space over (X, E) and τ is a soft topology over (X, E). The members of τ are called the soft open sets in (X, τ, E). The soft complements of soft open sets are known as soft closed sets. The soft interior and soft closure of soft set can be defined in the usual way. If $F\in$ S(X,E) then

*softInt*F = the soft interior of F in (X, τ, E) and *softCl*F = the soft closure of F in (X, τ, E).

Soft nearly open & soft nearly closed sets namely soft regular open, soft regular closed, soft α-open, soft α-closed, soft semi-open, soft semi-closed, soft pre-open, soft pre-closed, soft β-open, soft β-closed, soft b-open, soft b-closed, soft *b-open, soft *b-closed, soft $b^{\#}$-open, soft $b^{\#}$-closed, a soft q-set, a soft p-set, a soft Q-set are defined in[6,7]. The following operators will be used to define the above sets.

$\lambda(F) = softCl(softInt F)$.

$\mu(F) = softInt(softCl F)$.

$\gamma(F) = softInt(softCl(softInt F))$.

$\eta(F) = softCl(softInt(softCl F))$.

The next lemma, due to Shabir Hussain and Bashir Ahmad[13], gives the parametrized family of topologies induced by a soft topology.

1.3 Lemma

Let (X, τ, E) be a soft topological space. Then for each $e \in E$, the collection of sub sets F(e) of X, $F \in \tau$ is a topology on X, denoted by τ_e.

The family $\{\tau_e : e \in E\}$ is called a parametrized family of topologies induced by the soft topology τ. The next definition is useful in sequel.

1.4 Definition

A soft set F in a soft topological space (X, τ, E) is called

(i) soft regular open if $F = \mu(F)$ and soft regular closed if $F = \lambda(F)$,

(ii) soft α-open if $F \sqsubseteq \gamma(F)$ and soft α-closed if $\eta(F) \sqsubseteq F$,

(iii) soft semi-open if $F \sqsubseteq \lambda(F)$ and soft semi-closed if $\mu(F) \sqsubseteq F$,

(iv) soft pre-open if $F \sqsubseteq \mu(F)$ and soft pre-closed if $\lambda(F) \sqsubseteq F$,

(v) soft β-open if $F \sqsubseteq \eta(F)$ and soft β-closed if $\gamma(F) \sqsubseteq F$,

(vi) soft b-open if $F \sqsubseteq \mu(F) \sqcup \lambda(F)$ and soft b-closed if $\mu(F) \sqcap \lambda(F) \sqsubseteq F$,

(vii) soft *b-open if $F \sqsubseteq \mu(F) \sqcap \lambda(F)$ and soft *b-closed if $\mu(F) \sqcup \lambda(F) \sqsubseteq F$,

(viii) soft $b^{\#}$-open if $F = \mu(F) \sqcup \lambda(F)$ and soft $b^{\#}$-closed if $\mu(F) \sqcap \lambda(F) = F$,

(ix) a soft q-set if $\mu(F) \sqsubseteq \lambda(F)$,

(x) a soft p-set if $\lambda(F) \sqsubseteq \mu(F)$ and

(xi) a soft Q-set if $\mu(F) = \lambda(F)$.

Let $A \subseteq X$ and $e \in E$. The soft set A_e is called a quasi soft set over X if $A_e(e) = A$ and $A_e(\alpha) = \emptyset$ for every $\alpha \in E \setminus \{e\}$. It is easy to see that every soft point is a quasi soft set. The converse is not true. Let $QS(\tau_e) = \{A_e : A \in \tau\}$. Evanzalin[5] has proved some interesting properties of the soft interior and soft closure operators as given in the next lemma.

1.5 Lemma

Let (X, τ, E) be a soft topological space with $QS(\tau_e)$ is contained in τ for every $e \in E$

and $F \in S(X,E)$. Then for every $e \in E$

(i) $(softInt F)(e) = Int(F(e))$ and $(softCl F)(e) = Cl(F(e))$,

(ii) $(\lambda(F))(e) = Cl\,Int(F(e))$ and $(\mu(F))(e) = Int\,Cl(F(e))$,

(iii) $(\gamma(F))(e) = Int\,Cl\,Int(F(e))$ and

(iv) $(\eta(F))(e) = Cl\,Int\,Cl(F(e))$.

2. Soft near relation

2.1 Definition

Let F and $G \in S(X, E)$ and let (X, τ, E) be a soft topological space. F is soft near to G if $softInt\,F = softInt\,G$.

2.2 Lemma

Let (X, τ, E) be a soft topological space. The relation "is soft near to" is an equivalence relation on the soft sets over (X, E).

Proof: Let F, G, H be the soft sets over (X, E). Since *softInt* F = *softInt* F, F is soft near to F so that the relation is reflexive.

F is soft near to G $\Rightarrow$ *softInt* F = *softInt* G = *softInt* G = *softInt* F $\Rightarrow$ G is soft near to F.

F is soft near to G and G is soft near to H $\Rightarrow$ *softInt* F = *softInt* G and *softInt* G = *softInt* H

$\Rightarrow$ *softInt* F = *softInt* H

$\Rightarrow$ F is soft near to H.

The equivalence classes of the relation "is soft near to"are called the soft near classes of the soft subsets over (X, E). If F is a soft subset of X then the soft near class of F is *softnear* [F] = {G : F is soft near to G}.

2.3 Proposition

There is an one-to-one correspondence between soft topology τ on X and the collection of soft near classes,

Proof: For any soft open set G in (X, τ, E) , a soft subset F of X is soft near to G if and only if G= *softInt* F . Conversely every soft subset F of X is soft near to some soft open set . This proves the proposition.

2.4 Proposition

F$\in$ *softnear* [G] $\Leftrightarrow$ G$\in$ *softnear* [F].

2.5 Proposition

Let F be a soft subset of X over E.

(i) If F is soft semi-closed then F is soft near to *softCl*F.

(ii) If F is soft β-closed in (X, τ, E) then F is soft near to $\lambda(F)$.

Proof: Suppose F is soft semi-closed in (X, τ, E) . Then *softInt* F = $\mu(F)$ = *softInt*(*softCl*F) that implies F is soft near to *softCl*F. This proves (i). Suppose F is soft β-closed in (X, τ, E). Then $\gamma(F) \sqsubseteq F$ that implies *softInt* F $\sqsubseteq$ *softInt* (*softCl*(*softInt*F)) $\sqsubseteq$ *softInt* F so that

softInt F = *softInt* (*softCl*(*softInt*F)) that proves that F is soft near to $\lambda(F)$.

This proves (ii).

2.6 Corollary

(i) If F is soft regular open or soft α-closed then F is soft near to *softCl*F .

(ii) If F is soft pre-closed or soft b-closed or soft $b^{\#}$-closed or soft *b-closed then F is soft near to $\lambda(F)$.

Proof : Since every soft regular open soft set is soft semi-closed and

every soft α-closed soft set is soft semi-closed , the assertion(i) follows from

Proposition 2.5(i). Since soft pre-closed $\Rightarrow$ soft b-closed $\Rightarrow$ soft β-closed,

since soft $b^{\#}$-clod $\Rightarrow$ soft b-closed $\Rightarrow$soft β-closed and

since soft *b-closed $\Rightarrow$ soft b-closed $\Rightarrow$soft β-closed, the assertion (ii) follows from Proposition 2.5(ii).

2.7 Proposition

Let F_1 be soft near to G_1 and F_2 be soft near to G_2. Then

(i) $F_1 \sqcap F_2$ is soft near to $G_1 \sqcap G_2$ and

(ii) $F_1 \sqcap G_2$ is soft near to $G_1 \sqcap F_2$

Proof: Suppose F_1 is soft near to G_1 and F_2 is soft near to G_2. Then *softInt* F_1 = *softInt* G_1 and *softInt* F_2 = *softInt* G_2. Now *softInt* $(F_1 \sqcap F_2)$ = *softInt* $F_1 \sqcap$ *softInt* F_2 = *softInt* $G_1 \sqcap$ *softInt* G_2 = *softInt* $(G_1 \sqcap G_2)$

that implies $F_1 \sqcap F_2$ is soft near to $G_1 \sqcap G_2$. This proves (i) and the proof for (ii) is analogous.

2.8 Definition

Let $F \in$ *softnear* [G]. If $F \sqsubseteq G$ then F is called a soft near soft subset of G and if $F \sqsupseteq G$ then F is called a soft near soft superset of G.

2.9 Proposition

F is a soft near soft subset of G $\Leftrightarrow$ G is a soft near soft super set of F.

Proof: F is a soft near soft subset of G $\Leftrightarrow$ $F \in$ *softnear* [G] and $F \,{}^{s}\!\sqsubseteq G$

$\Leftrightarrow$ $G \in$ *softnear*[F] and $G \,{}^{s}\!\sqsupseteq F$

$\Leftrightarrow$ G is a soft near soft super set of F.

2.10 Proposition

Every soft near soft subset of soft open soft set is soft open.

Proof: Let F be a soft near soft subset of G and G be soft open.

Then $F \sqsubseteq G$ = *softInt* F = *softInt* G that implies $F \sqsubseteq G$ = *softInt* G = *softInt* F so that

F= *softInt* F is soft open.

2.11 Proposition

Let F be soft near soft subset of G. Then F is soft semi-open or soft α-open according as G is soft semi-open or soft α-open.

Proof: Suppose G is soft semi open. Then $G \sqsubseteq \lambda(G)$. Since G is soft near to F,

*Int*G = *Int*F that implies $F \sqsubseteq G \sqsubseteq \lambda(G)$ = *softCl*(*softInt*G) = *softCl*(*softInt*F). This proves that F is soft semi- open. If G is soft α-open then $G \sqsubseteq \gamma(G)$= *softInt* (*softCl*(*softInt*G)) that implies $F \sqsubseteq G \sqsubseteq \gamma(G)$= *softInt* (*softCl*(*softInt*G)) = *softInt* (*softCl*(*softInt*F)) so that proving that F is soft α-open.

2.12 Corollary

(i) F is soft semi open $\Leftrightarrow$ every soft near soft subset of F is soft semi open.

(ii) F is soft α-open $\Leftrightarrow$ every soft near soft subset of F is soft α-open.

2.13 Proposition

Let F be a soft near soft super set of G. Then F is soft pre closed or soft β-closed according as G is soft pre closed or soft β-closed.

Proof: Suppose G is soft pre closed. Then $\lambda(G)$ = *softCl*(*softInt*G) $\sqsubseteq$ G.

Since F is soft near to G, *softInt* G = *softInt* F that implies

$\lambda(F)$ = *softCl*(*softInt*F) =$\lambda(G)$ = *softCl*(*softInt*F) $\sqsubseteq G \sqsubseteq F$. This proves that F is soft pre closed. If G is soft β-closed then $\gamma(G)$= *softInt* (*softCl*(*softInt*G)) $\sqsubseteq$ G that implies

γ(F)= *softInt* (*softCl*(*softInt*F)) = *softInt* (*softCl*(*softInt*G)) $\sqsubseteq$ G $\sqsubseteq$F that shows that

F is soft β-closed.

2.14 Corollary

(i) F is soft pre closed $\Leftrightarrow$ every soft near soft super set of F is soft pre closed.

(ii) F is soft β-closed $\Leftrightarrow$ every soft near soft super set of G is soft β-closed.

2.15 Proposition

Let (X, τ, E) be a soft topological space such that QS(τ_e) is contained in τ for every e$\in$E and F , G $\in$S(X,E). Then for every e$\in$E, F is soft near to G if and only if F(e) is near to G(e) in (X, τ_e).

Proof: F is soft near to G if and only if *softInt* F = *softInt* G. Then by applying Lemma 1.4 we see that F is soft near to G $\Leftrightarrow$ (*softInt* F)(e) = (*softInt* G)(e)

$\Leftrightarrow$ *Int*(F(e)) = *Int*(G(e))

$\Leftrightarrow$ F(e) is near to G(e)

2.16 Lemma

Let (X, τ, E) be a soft topological space such that QS(τ_e) is contained in τ for every e$\in$E and A $\subseteq$X. Then for every e$\in$E, (*softInt* A_e)(e) = *Int*A and

(*softInt* A_e)(α) = $\varnothing$ for every $\alpha \neq$e.

2.17 Proposition

Let (X, τ, E) be a soft topological space with QS(τ_e) is contained in τ for every e$\in$E and A, B $\subseteq$X. Then for every e$\in$E, A_e is soft near to B_e if and only if A is near to B in (X, τ_e).

Proof: A_e is soft near to B_e $\Leftrightarrow$ (*softInt* A_e)(α) =(*softInt* B_e)(α) for every α

$\Leftrightarrow$ (*softInt* A_e)(e) = (*softInt* B_e)(e) and

(*softInt* A_e)(α) = (*softInt* B_e)(α) for $\alpha \neq$e

$\Leftrightarrow$ *Int*A = *Int*B and (*softInt* A_e)(α) = (*softInt* B_e)(α) = $\varnothing$ in (X, τ_e) for $\alpha \neq$e

$\Leftrightarrow$ A is near to B in (X, τ_e).

Conclusion

Soft subsets of a soft topological space are classified by using the soft near relation. This relation is also characterized by using the near relation in topological spaces.

References

[1] Akbar Tayebi, E. Peyghan and B. Samadi, About soft topological spaces, *J. New Results in Science*, 2(2013), 60-75.

[2] Bashir Ahmad and Sabir Hussain, On some structures of soft topology, *Mathematical Sciences*, (2012), 1-7.

[3] Chang Wang and Yaya Li, Topological Structure of vague soft sets, *Abstract and Applied Analysis*, (2014), 8 pages.

[4] Chitralekha , M. Anitha and N. Meena , 'Near and closer relations in topology' Malaya journal of matematik, 8(4)(2020), 2169-2172.

[5] Evanzalin Ebenanjar .P and Thangavelu.P, Between Nearly open sets and Soft Nearly open sets, *Appl. Math. Inf. Sci*. 10(6)(2016), 1-5.

[6] Kiruthika.M , A new look between soft topology and its induced topologies , Ph.D thesis, Bharathiar university, Coimbatore, TN, India.(2019).

[7] Kiruthika.M, Thangavelu.P, Soft $b^{\#}$ - open sets, *International Journal of Applied Engineering Research*, 11(1) (2016), 529-532.

[8] Molodtsov.D , Soft Set Theory – First Results, *Computers and Mathematics with Applications*, 37(1999), 19-31.

[9] Muhammad Shabir & Munazza Naz 2011,' On Soft Topological Spaces', Comput. Math. Appl., 61(2011), 1786-1799.

[10] Naim Cagman, Serkan Karatas, Serdar Enginoglu, Soft topology, *Computers and Mathematics with Applications*, 62(2011), 351-358.

[11] Peyghan.E , Samadi.B , Tayebi.A , About soft topological spaces, *Journal of New Results in Science*, 2(2013), 60-75.

[12] Ping Zhu and Qiaoyan, Operations on soft sets Revisited, *Journal of Applied Mathematics*, (2013), 7-pages http://dx.org/1010.1155/2013/105752.

[13] Rohini Devi .P, Padmapriya.R , Characterization of soft sets having the same soft closer in soft topology, *Sylwan*, 166(2022), 59-69.

[14] Shabir Hussain and Bashir Ahmad, Some properties of soft topological spaces, *Computers and Mathematics with Applications*, 62(2011), 4058-4067.

[15] Thamizharasi.G , 'Studies in Bitopological Spaces', Ph.D thesis, Manonmaniam Sundaranar university, Tirunelveli , TN, India.(2010).