DOI : https://doi.org/10.56452/5-2

ISSN: 0974-5823 Vol. 5 No. 2 December, 2020

International Journal of Mechanical Engineering

Discuss and Analyze on Linear Approximations

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Abstract:

This study describes the metacognition profile of mathematics and mathematics and mathematics education students in understanding the concept of differential calculus. The metacognition profile is a natural and in fact description of a person is cognition that involves his own thinking in terms of using his knowledge, planning and monitoring his thinking process, and evaluating his thinking results when understanding a concept. The purpose this study is to produce the metacognition profile of mathematics and mathematics education students in understanding the concept of differential calculus. This research method is explorative method with the qualitative approach. In this note there are so many examples of calculus which are related to our life.

Keywords:- Derivatives, first order derivatives, biology, non-linear mathematics, etc.

INTRODUCTION:-

Calculus was developed independently by English physicist and mathematician Sir Isaac Newton (1642–1727) and German mathematician Gottfried Wilhelm Leibniz (1646–1716) around the middle part of the seventeenth century. Newton was a physicist as well as a mathematician. He found that the mathematics of his time was not sufficient to solve the problems he was interested in, so he invented new mathematics. About the same time, another mathematician, Leibniz, developed the same ideas as Newton. Newton was interested in calculating the velocity of an object at any instant. For example, if a person sits under an apple tree, as legend has it Newton did, and an apple falls and hits the person's head, that person might ask how fast the apple was traveling just before impact. More importantly, many of today's scientists are interested in calculating the rate at which a satellite's position changes with respect to time (its rate of speed). Most investors are interested in how a stock's value changes with time (its rate of growth). In fact, many of today's important problems in the fields of physics, chemistry, engineering, economics, biology, and the other sciences involve finding the rate at which one quantity changes with respect to another, that is, they involve finding the derivative.(see[1-7]).

The radar speed gun was invented by John L. Barker Sr., and Ben Midlock, who developed radar for the military while working for the Automatic Signal Company (later Automatic Signal Division of LFE Corporation) in Norwalk, CT during World War II. Originally, Automatic Signal was approached by Grumman Aircraft Corporation to solve the specific problem of terrestrial landing gear damage on the now-legendary PBY Catalina amphibious aircraft. Barker and Midlock cobbled a Doppler radar unit from coffee cans soldered shut to make microwave resonators. The unit was installed at the end of the runway (at Grumman's Bethpage, NY facility), and aimed directly upward to measure the sink rate of landing PBYs. After the war, Barker and Midlock tested radar on the Merritt Parkway. In 1947, the system was tested by the Connecticut State Police in Glastonbury, Connecticut, initially for traffic surveys and issuing warnings to drivers for excessive speed. Starting in February 1949, the state police began to issue speeding tickets based on the speed recorded by the radar device. In 1948, radar was also used in Garden City, New York (see[1 - 7]).

Calculus: -The branch of mathematics that deals with the finding and properties of derivatives and integrals of functions, by methods originally based on the summation of infinitesimal differences.

Derivative: -By the geometrical approach: The slope of the curve for the given function is called the derivative of a function. By physical approach: The instantaneous rate of change of a function concerning the variable at a point is called the derivative of a function.

Radar gun:- A handheld device used by traffic police to estimate the speed of a passing vehicle.

DISCUSSION:-

Types of Derivatives

First and second-order derivatives are two types of derivatives categorised based on their order. These can be described as follows:-

Derivatives of First-Order

The first order derivatives show whether the function is going up or down, so they show which way the function is going. The first derivative, also known as the first-order derivative, is a rate of change that occurs instantly. The slope of the tangent line can also be used to anticipate it.

Derivatives of Second-Order

Second-order derivatives are used to figure out what the graph of a given function looks like. Concavity can be used to classify the functions. The concavity of a graph function can be divided into two categories:

- Concave up
- Concave Down

Real-World Applications of Derivatives

- To calculate the profit and loss in business using graphs.
- To check the temperature variation.
- To determine the speed or distance covered such as miles per hour, kilometre per hour etc.
- Derivatives are used to derive many equations in Physics.
- In the study of Seismology like to find the range of magnitudes of the earthquake.
- The pace at which a population (whether a group of humans or a colony of bacteria) grows over time, can be used to forecast population size changes soon
- Temperature variations as a function of location can be used to forecast weather
- Stock market fluctuations throughout time can be used to forecast future stock market behaviour
- Automobiles
- An odometer and a speedometer are always present in a car. These two gauges operate together to give the driver information about his speed and distance travelled
- A radar gun can determine the automobile’s speed and report the distance the car was from the radar gun by using a derivative

Derivative are being used in our life for serving lots of purpose whether we know it or we don’t. It almost covers all the branches of study such as physics, chemistry, mathematics, biology, statistics, etc.

Applications of Derivatives in Mathematics

Derivatives is widely used in mathematics for solving different problems. It is mainly used in calculus. Some of the applications of derivative in mathematics are briefly discussed below: -

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Rate of Change of a Quantity

This is the general and most important application of derivative. For example, to check the rate of change of the volume of a cube with respect to its decreasing sides, we can use the derivative form as dy/dx. Where dy represents the rate of change of volume of cube and dx represents the change of sides of the cube.

Increasing and Decreasing Functions

To find that a given function is increasing or decreasing or constant, say in a graph, we use derivatives. If f is a function which is continuous in [p, q] and differentiable in the open interval (p, q), then,

F is increasing at [p, q] if $f'(x) > 0$ for each $x \in (p, q)$

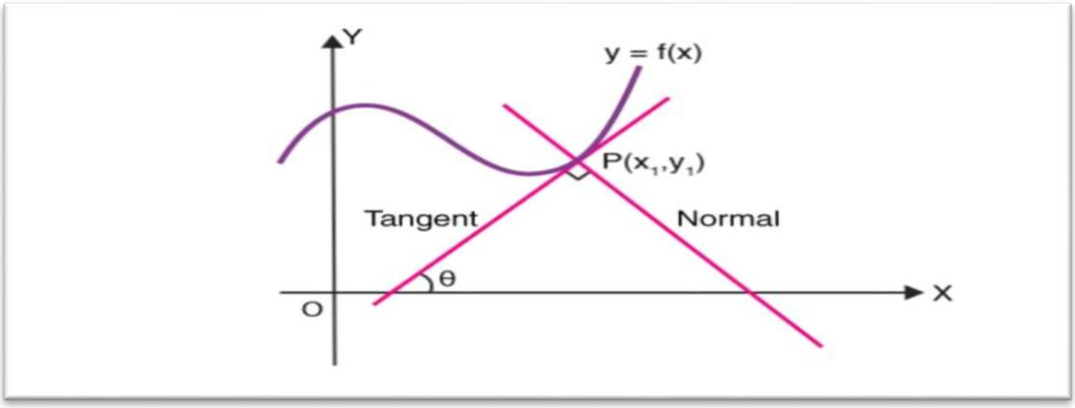
F is decreasing at [p, q] if $f'(x) < 0$ for each $x \in (p, q)$

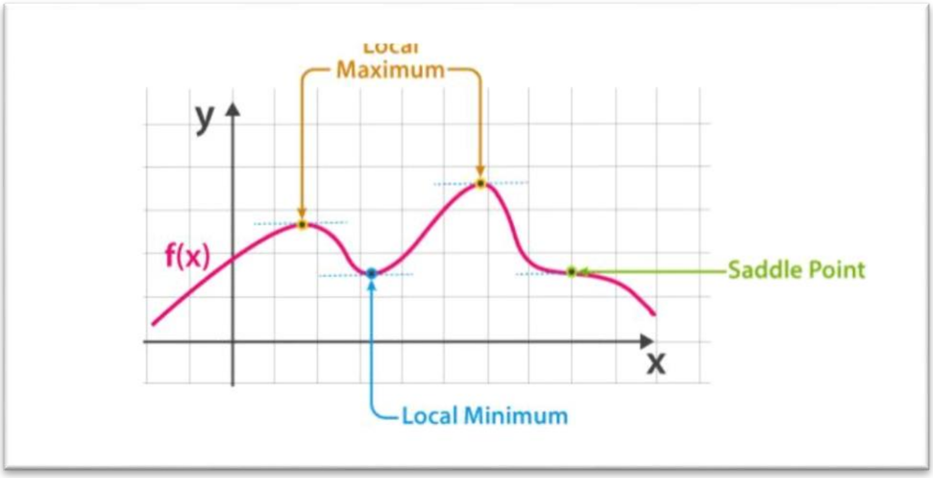
F is constant function in [p, q], if $f'(x)=0$ for each $x \in (p, q)$

Tangent and Normal To a Curve

A tangent is a line that touches the curve at a point and doesn't cross it, whereas normal is perpendicular to that tangent.

Let the tangent meet the curve at $P(x_1, y_1)$.





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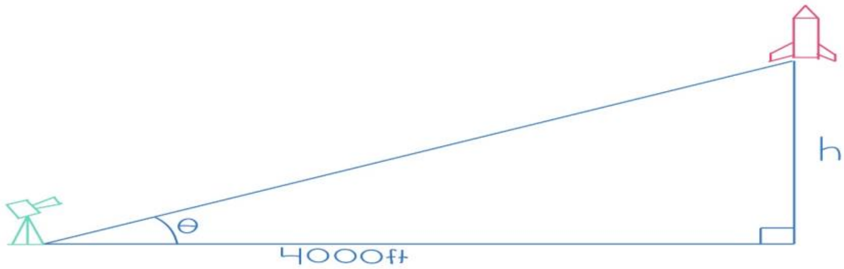
APPLICATIONS OF DERIVATIVE IN REAL LIFE EXAMPLES

Problem 1

Your camera is set up 4000 ft From a rocket launch pad. The rocket launches, and when it reaches an altitude of 1500 ft Its velocity is 500 ft/s. What rate should your camera's angle with the ground change to allow it to keep the rocket in view as it makes its flight?

Solution

1. Sketch the problem.



Your camera is 4000 feet from the launchpad of rocket. The two related rates -the angle of our camera (θ) and the height (h)

Of the rocket-are changing with respect to time (t).

Here, θ is the angle between your camera lens and the ground and his height of the rocket above the ground.

2.Clarify what exactly we are trying to find.

To find the rate of change of our cameras angle to the ground when the rocket is 1500 feet above the ground. Both of these variables are changing with respect to time.

This means we need to find $\frac{d\theta}{dt}$ when h = 1500 feet. We also know that the velocity of the rocket at that time is $\frac{dh}{dt}$ = 500 ft/s

3.Determine what equation relates the two quantities h and θ

(i) looking back at our picture in step 1, we think about using a trigonometric equation. What relates the opposite and adjacent sides of a right triangle? The tan function: so we have:

tan θ =h/4000

(ii) rearing to solve for h gives,

h= 4000tan(theta)

4.Differentiate this to get,

$(dh/dt)=4000\sec^2\theta \frac{d\theta}{dt}$

5. Find $(\frac{d\theta}{dt})$ when h=1500 feet

(i) To find $(\frac{d\theta}{dt})$, we first need to find $\sec^2(\theta)$.How can we do that?

a) going back to trigonometry, you know that

$\sec\theta=\frac{h}{b}$.

b) And , from the givens in this problem, we know that adjacent =4000 feet and opposite=h=1500 feet

c)So, you can use the Pythagorean theorem to solve for hypotenuse.

$a^2+b^2=c^2$

Or, $(4000)^2+(1500)^2=(\text{hypotenuse})^2$

Hypotenuse=500√3 feet

d)Therefore, when

h=1500 feet

$sec^2(\theta)$ is: -

$sec^2(\theta)$ =(hypotenuse/adjacent) ^{2}= (500√73/4000) ^{2}=73/64

(ii)Plug in the values for $sec^2(\theta)$ and (dh/dt) into the function

∴ (dh/dt) =4000$sec^2(\theta)(\frac{d\theta}{dt})$

500=4000(73/64) $(\frac{d\theta}{dt})$

:. $(\frac{d\theta}{dt})$=8/73

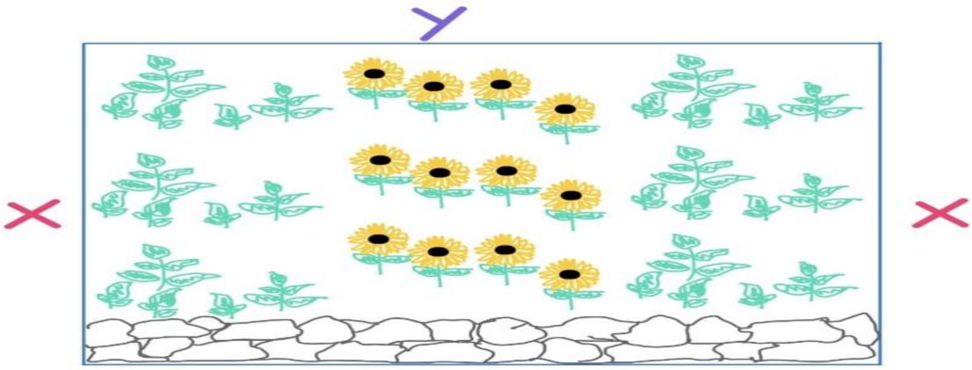
6.Therefore, the rate that our cameras angle with the ground should change to allow it to keep the rocket you was it makes it flight is

$(\frac{d\theta}{dt})$=8/73 rd/s

Problem 2

You are an agricultural engineer, and you need to fence a rectangular area of some farmland. One side of the space is blocked by a rock wall, so you only need fencing for three sides. Given that you only have 1000 feet of fencing, what are the dimensions that would allow you to fence the maximum area? What is the maximum area?

Determine the dimensions X and Y That will maximize the area of the farmland using 1000 feet of fencing.



1.Let X be the length of the sides of the farmland that run perpendicular to the rock wall, and let Y be the length of the side of the farmland that runs parallel to the rock wall. Then the area of the farmland is given by the equation for the area of a rectangle:

A=X.Y

2.since you want to find the maximum possible area given the constraint of 1000 feet of fencing to go around the perimeter of the farmland, you need an equation for the perimeter of the rectangular space.

(i) don't forget to consider that the French only needs to go around 3 of the sides:

so, your constraint equation is: 2X+Y=1000

(ii) Now you want to solve this equation for Y so that you can rewrite the area equation in terms of X only: Y=1000-2X

(iii) Rewriting the area equation, you get: A=X.Y=X.(1000-2X) =1000X-2X^2

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3.Before jumping right into maximizing the area you need to determine what your domain is

(I)First, you know that the lengths of the sides of your farmland must be positive ,i.e, X and Y can't be negative numbers.

a) Since, Y=1000-2X and you need X>0 and Y>0 then when you solve for X, you get:X=(1000-Y)/2

b) Minimizing Y, i.e., if Y=1, you know that: X<500

c) so, you need to determine the maximum value of A(X) for X on the open interval of (0,500) -how ever, you don't know that a function necessarily has a maximum value on an open interval, but you do know that a function does have a maximum and minimum value on a closed interval. Therefore, you need to consider the area function $A(X)=1000X-2X^2$ over the closed interval of [0,500].

4.Find the maximum possible area of the farmland by maximizing $A(X) = 1000X-2X^2$ over the closed interval of [0,500].

(I) Since A(X) is a continuous function on a closed, bounded interval you know that, by the extreme value theorem, it will have maximum and minimum values. These extreme values occur at the end points and critical points.

-At the end points, you know that A(X)=0.

-since the area must be positive for all values of X in the open interval of (0,500), the maximum most occur at a critical point. To find critical points you need to take the first derivative of A(X), CTET equal to zero and solve for X.

$A(X)=1000X-2X^2$

$A'(X)= 1000-4X$

$0=1000-4X$

$X=250$

-the only critical point is X=250. Therefore, the maximum area must be when X=250.

a) login this value into a perimeter equation, you get the Y-value of this critical point:

$Y=1000-2X=1000-2(250)=500$

b) Therefore, to maximize the area of farmland, X=250 feet and the area is125000 feet².

Problem 3

You are the chief financial officer at a rental car company. You found that if you charge your customers P dollars per day to rent a car, where 20<P<100, the number of cars N that your company rent per day can be modeled using the linear function N(P)=600-6P. If the company charges $20 or less per day, they will rent all of their cars. If the company charges $100 per day or more, they won't rent any cars.

How what should you tell the Honours of the company to rent the cars to maximize revenue?

Solution:-

1.Let P be the price charged per rental car per day. Let N be the number of cars your company rents per day. Let R be the revenue earned per day.

2. Find an equation that relates all three of these variables.

-revenue earned per day is the number of cars rented per day times the price charged per rental car per day: $R=N\times P$

3. substitute the value of N has given in the original problem.

$R=N\times P=(600-6P)\times P=-6P^2+600P$

4.Determine what you domain is,

-since you intend to tell the owners to charge between $20 and $100 per car per day, you need to find the maximum revenue for P on the closed interval of [20,100].

5. Find the maximum possible revenue by maximizing $R(P)=-6P^2+600P$ over the closed

interval of [20,100].

-Since R(P) is a continuous function of closed, bounded interval, you know that, by the extreme value theorem, it will have maximum and minimum values. These extreme values occurred the end points and any critical points.

-find the critical points by taking the first derivative, setting it equal to zero, and solving for P.

$R(P)=-6P^2+600P$

$R'(P)=-12P+600$

$0=-12P+600$

$P=50$

-the only critical point is P=50.Therefore, the maximum revenue must be when P=50.

-plugging these value into your revenue equation, you get the R(P)-value of this critical point:R (P)=$-6P^2+600P$

$R(50)=-6(50)^2+600(50)=15000$

→Therefore to maximize revenue, you should tell the owners to charge $50 per car per

day.

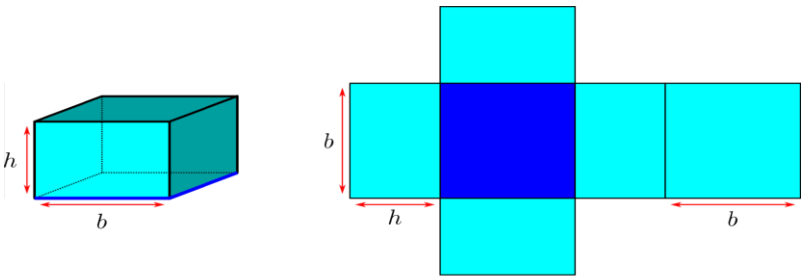
Problem 4

A closed rectangular container with a square base is to be made from two different materials. The material for the base costs $5 per square meter, while the material for the other five sides costs $1 per square meter. Find the dimensions of the container which has the largest possible volume if the total cost of materials is $72.

SOLUTION

We can follow the steps to find the solution.

- We need to determine the area of the two types of materials used and the corresponding total cost.
- Draw a picture of the box.



The more useful picture is the unfolded box on the right.

- In the picture we have already introduced two variables. The square base has side-length *b* metres and it has height *h* metres. Let the area of the base be A*b* and the area of the other fives sides be A*s* (both in m2), and the total cost be *C* (in dollars). Finally let the volume enclosed be Vm^3.
- Some simple geometry tells us that

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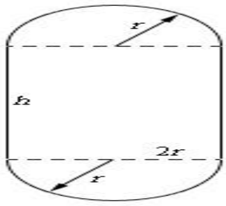
$Ab=b^2$
$As=4bh + b^2$
$V=b^2h$
$C=5.Ab + 1.As=5b^2+4bh+b^2=6b^2+4bh$

- To eliminate one of the variables we use the fact that the total cost is \$72.
  $C=6b^2+4bh=72$
  $4bh=72-6b^2$
  $h=(72-6b^2)/4b=(3/2).(12-b^2)/b$
- Substituting this into the volume gives
  $V=b^2h=(3b/2)(12-b^2)=18b-(3/2)b^3$

Now note that since b is a length it cannot be negative, so $b \geq 0$. Further since volume be negative, we must also have

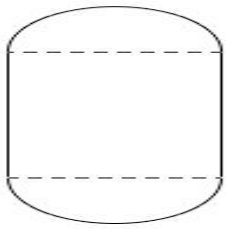


$12-b^2 \geq 0$
And so, $b \leq \sqrt{12}$

- Now we can apply Corollary 3.5.13 on the above expression for the volume with $0 \leq b \leq \sqrt{12}$. *The endpoints give:*
  $V(0)=0$
  $V(\sqrt{12})=0$
  The derivative is
  $V'(b)=18-(9b^2/2)$
  Since this is a polynomial there are no singular points. However we can solve $V'(b)=0$ to find critical points:
  $18-(9b^2/2)=0$ (divide by 9 and multiply by 2)
  $4-b^2=0$
  Hence $b=\pm 2$.
  Thus, the only critical point in the domain is $b=2$.
  The corresponding volume is
  $V(2)=18\times 2-(3/2)\times 2^3=36-12=24$.
  So by Corollary 3.5.13, the maximum volume is when 24 when b=2
  And
  $h=(3/2)\cdot(12-b^2)/b=(3/2).(12-4)/2=6$.
- All our quantities make sense; lengths, areas and volumes are all non-negative.
- Checking the question again, we see that we are asked for the dimensions of the container (rather than its volume) so we can answer with the container with dimensions 2×2×6m will be the largest possible.

Problem 5

We want to construct a window whose middle is a rectangle and the top and bottom of the window are semi-circles. If we have 50 meters of framing material what are the dimensions of the window that will let in the lightest?



Solution

STEP 1

Let's start with a quick sketch of the window.

Next, we need to set up the constraint and equation that we are being asked to optimize.

We are told that we have 50 meters of framing material (i.e. the perimeter of the window) and so that will be the constraint for this problem.

$50=2h+2(\pi r)=2h+2\pi r$

We are being asked to maximize the amount of light being let in and that is simply the enclosed area or,

$A=h(2r)+2[(1/2)\pi r^2]=2hr+\pi r^2$

With both of these equations we were a little careful with the last term. In each case we needed either the perimeter or area of each semicircle and there were two of them. The end result of course is the equation of the perimeter/area of a whole circle, but we really should be careful setting these equations up and note just where everything is coming from.

STEP 2

Now, let's solve the constraint for *h.*

$h=25-\pi r$

Plugging this into the area function gives,

$A(r)=2(25-\pi r)r+\pi r^2=50r-\pi r^2$

STEP 3

Finding the critical point(s) for this shouldn't be too difficult at this point. Here is the derivative.

$A'(r)=50-2\pi r$

From this it looks like we get a single critical points :

$r=25/\pi=7.9577$

STEP 4

The second derivative of the volume function is,

$A''(r)=-2\pi-2\pi$

From this we can see that the second derivative is always negative. Therefore

A(r) Will always be concave down and so the single critical point we got in Step 3 must be a relative maximum and hence must be the value that allows in the maximum amount of light.

STEP 5

Now, let's finish the problem by getting the radius of the semicircles.

$h=25-\pi(25/\pi)=0$

Okay, what this means is that in fact the most light will come from not even having a rectangle between the semicircles and just having a circular window of radius $r=25/\pi$.

Problem 6

A scientist in B- grade science fiction film is studying a sample of the rare and fictitious element., implasusium. With great effort he has produced a sample of pure implausium. The next day-17 hours later-he comes back to hours later-he comes back to his lab and discovers that his sample is now only 37% pure. What is the half life of element?

Solution:- Let $\theta\ (t)= \theta(0).e^{-kt}$

we also know that

θ (17)= 0.37 $\theta(0)$

θ (17)= 0.37 $\theta(0)$ = $\theta(0)$.e^{-17k}

=0.37 = e^{-17k}

K= 0.05849

Thus $t_{\frac{1}{2}} = \frac{log2}{k} \approx 11.85\ hours.$

CONCLUSION:- Calculus is one of those topics in mathematics where the algorithmic manipulation of symbols is easier than understanding the underlying concepts. Around 1680 Leibnitz invented a symbol system for calculus that codifies and simplifies the essential elements of reasoning. This paper is a guide to the advanced placement program in calculus for grade 12 in the MIT schools in Janakpurdham Nepal.

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